Project: admcycles
Path: GrushevskyEffective.ipynb
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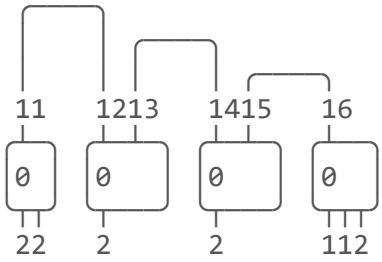
Kernel: SageMath 9.8


```
from admcycles.DR.graph import *
from admcycles.DR.relations import *
from admcycles.admcycles import Graphtodecstratum

L = all_pure_strata(0, 3, (1,1,2,2,2,2,2))  # list of all 1-dim strata in
Mbar_{0,7}/S_2 x S_5
M = [Graphtodecstratum(G) for G in L]       # convert to more readable format

# below are drawings of the 24 relevant graphs
prod(gam.gamma._unicode_art_() for gam in M)
```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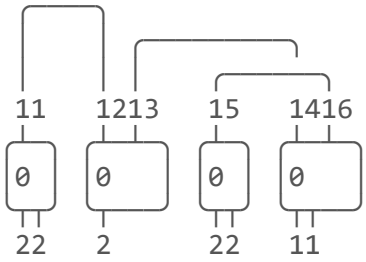
0 0 0 0
22 2 2 112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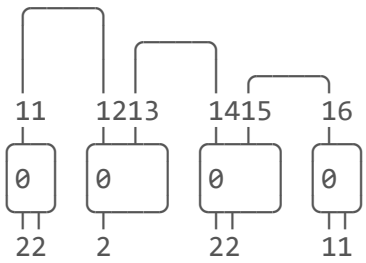
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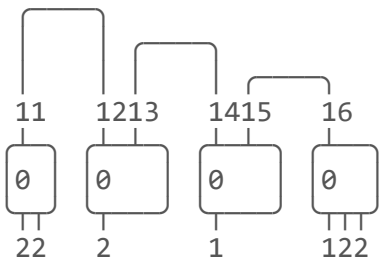
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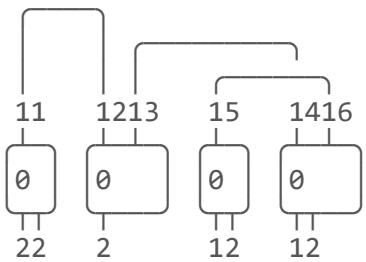
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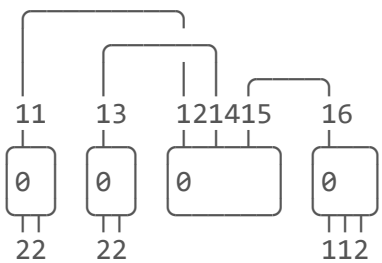
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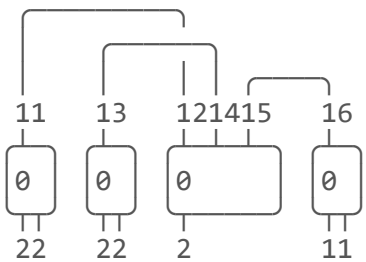
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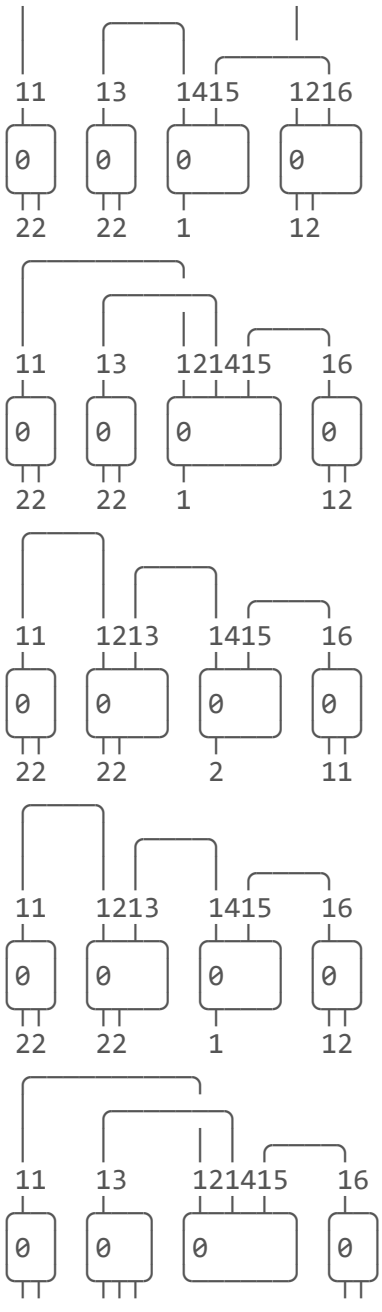
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0 0 0 0
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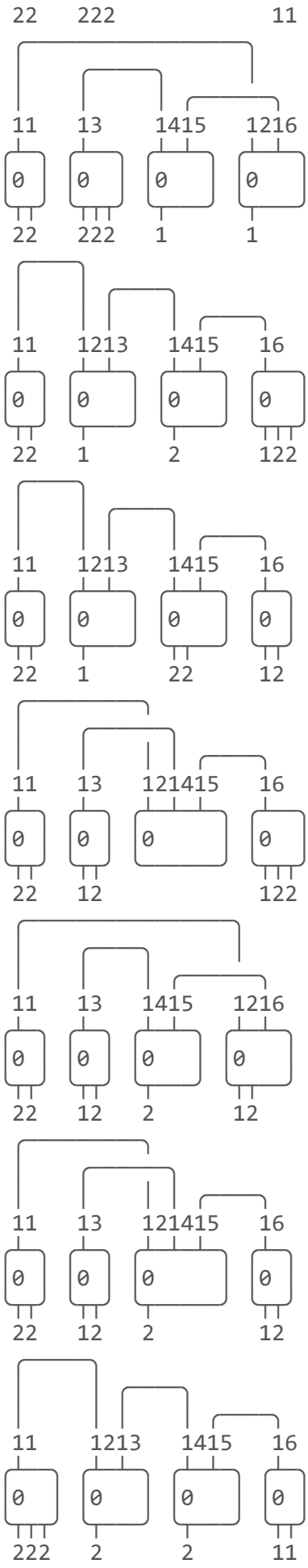
0 0 0 0
22 22 1 12
11 13 121415 16
0 0 0 0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11 13 1415 1216
0 0 0 0
22 222 1 1
11 1213 1415 16
0 0 0 0
22 1 2 122
11 1213 1415 16
0 0 0 0
22 1 22 12
11 13 121415 16
0 0 0 0
22 12 122
11 13 1415 1216
0 0 0 0
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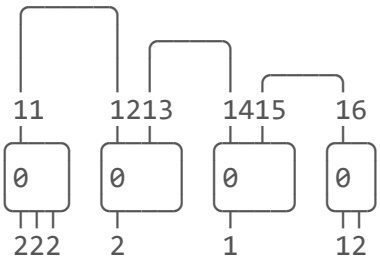
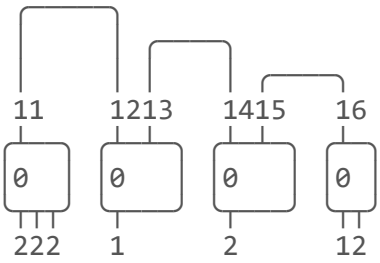
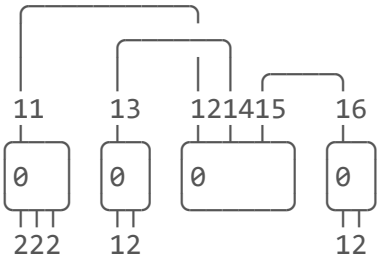
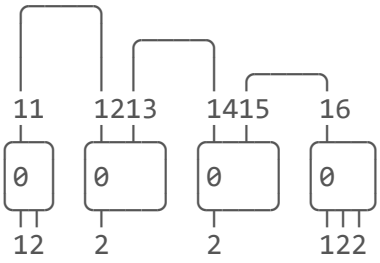
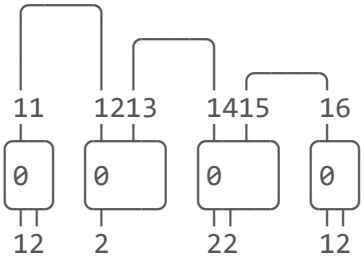
11 13 121415 16
0 0 0 0
22 12 2 12
11 1213 1415 16
0 0 0 0
222 2 2 11











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```
len(L)
```

24

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```
Rel = list_all_FZ(0, 3, (1,1,2,2,2,2,2))  # computing all tautological relations
between strata (decorated by kappa, psi)
```

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```
len(Rel)
```

```
1102
```

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```
RelMat = matrix(QQ, Rel); RelMat
```

```
1102 x 250 dense matrix over Rational Field (use the '.str()' method to see the
entries)
```

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```
RelMat.rank()  # matrix of relations has 250 columns (corresponding to decorated
strata) and rank 245, so rank H_2 = 5
```

```
245
```

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```
RelV = span(QQ, Rel)                # space of relations
Amb = RelV.ambient_vector_space()   # space of decorated strata
H2 = Amb.quotient(RelV); H2         # H_2 = Strata / Relations
```

```
Vector space quotient V/W of dimension 5 over Rational Field where
V: Vector space of dimension 250 over Rational Field
W: Vector space of degree 250 and dimension 245 over Rational Field
Basis matrix:
245 x 250 dense matrix over Rational Field
```

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```
# convert the 24 undecorated strata above into vectors
# below we see that these are just the last 24 entries of the list of all decorated
strata
num_strata = [num_of_stratum(G, 0, 3, (1,1,2,2,2,2,2)) for G in L]; num_strata
```

```
[226,
 227,
 228,
 229,
 230,
 231,
 232,
 233,
 234,
 235,
 236,
 237,
 238,
 239,
 240,
 241,
 242,
 243,
 244,
 245,
 246,
 247,
 248,
 249]
```

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```
strata_vecs = [Amb.basis()[v] for v in num_strata]  # get the vectors corresponding
to the 24 pure strata
```

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```
u = strata_vecs[0]
```

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```
H2(u) # this gives us the representation of the first graph in our list with respect
to a fixed basis of H_2
```

```
(-2/5, 7/50, -1/10, -1/25, 1/20)
```

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```
N = matrix(QQ, [H2(u) for u in strata_vecs])  # converting all 24 graphs into this
fixed basis
for n in N:
    print(n)
```

```
(-2/5, 7/50, -1/10, -1/25, 1/20)
(-5/2, 7/20, 1/4, -1/10, 3/40)
(-3/10, -7/100, 7/20, 1/50, -3/40)
(7/10, -7/100, -1/20, 1/50, -1/40)
(-7/5, 7/50, 3/10, -1/25, 0)
(-2/5, 7/50, -1/10, -1/25, 1/20)
(3/5, -4/25, -1/5, 7/75, -1/10)
(1/10, -1/100, -3/20, 2/75, -1/40)
(-2, 1/5, 1/5, -1/30, 0)
(6/5, -3/25, -2/5, 4/75, 0)
(6/5, -3/25, -2/5, 4/75, 0)
(3/10, -3/100, 3/20, -1/50, 1/40)
(3/10, -3/100, 3/20, -1/50, 1/40)
(7/10, -7/100, -1/20, 1/50, -1/40)
(-3/10, 3/100, 1/20, -1/75, 1/40)
(7/10, -7/100, -1/20, 1/50, -1/40)
(1/10, -1/100, -3/20, 2/75, -1/40)
(-9/10, 9/100, -1/20, -1/150, 1/40)
(3/10, -3/100, 3/20, -1/50, 1/40)
(3/10, -3/100, 3/20, -1/50, 1/40)
(3/10, -3/100, 3/20, -1/50, 1/40)
(3/10, -3/100, 3/20, -1/50, 1/40)
(7/10, -7/100, -1/20, 1/50, -1/40)
(-3/10, 3/100, 1/20, -1/75, 1/40)
```

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```
pivs = N.transpose().pivots(); pivs  # this tells us that entries (0, 1, 2, 6, 9) of our list form a basis of H_2
```

```
(0, 1, 2, 6, 9)
```

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```
Nsub = matrix(QQ, [H2(strata_vecs[u]) for u in pivs])
Nsub.rank()
```

```
5
```

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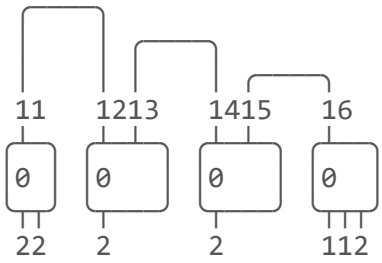
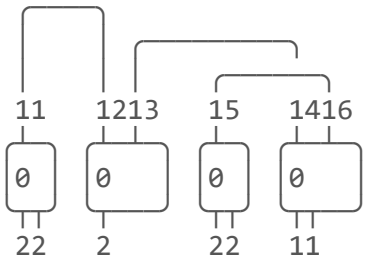
```
prod(M[u]._unicode_art_() for u in pivs)  # the 5 graphs below form a basis
```

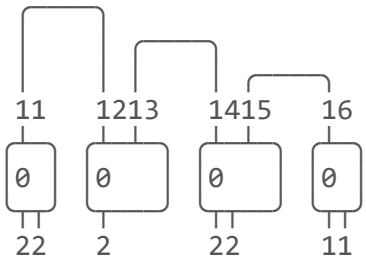
Graph :



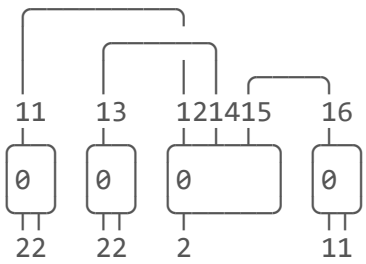
Polynomial : 1
Graph :



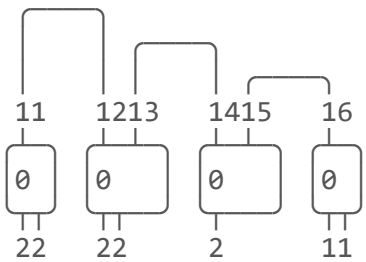
Polynomial : 1
Graph :



Polynomial : 1
Graph :



Polynomial : 1
Graph :



Polynomial : 1

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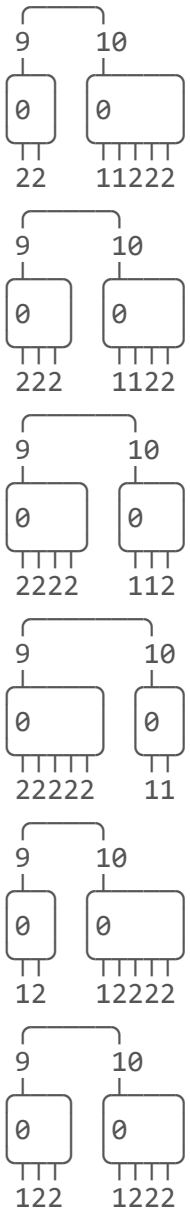
```
Divs = all_pure_strata(0, 1, (1,1,2,2,2,2,2))  # list of all codim 1 strata in
Mbar_{0,7}/S_2 x S_5
MDivs = [Graphtodecstratum(G) for G in Divs]       # convert to more readable format
prod(gam.gamma._unicode_art_() for gam in MDivs)
```



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```
len(Divs)
```

6

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```
num_div_strata = [num_of_stratum(G, 0, 1, (1,1,2,2,2,2,2)) for G in Divs];
num_div_strata
```

```
[3, 4, 5, 6, 7, 8]
```

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```
pairmat = matrix(QQ, pairing_submatrix(tuple(num_strata), tuple(num_div_strata), 0,
3, (1,1,2,2,2,2,2)))
print(pairmat.str())
```

```
[   0    0  -48  240   48    0]
[ -24  -24    0  240    0   24]
[  24  -24   96 -240    0    0]
[  24    0    0    0   48  -12]
[   0  -24   48    0    0   12]
[   0    0  -48  240   48    0]
[ -48   48   48 -240    0    0]
[ -24   24    0    0   24    0]
[ -48    0   48    0  -24   24]
[   0   48  -48    0    0    0]
[   0   48  -48    0    0    0]
[  72  -24    0    0    0    0]
[  72  -24    0    0    0    0]
[  24    0    0    0   48  -12]
[  24    0    0    0  -24   12]
[  24    0    0    0   48  -12]
[ -24   24    0    0   24    0]
[ -24   24    0    0  -48   24]
[  72  -24    0    0    0    0]
[  72  -24    0    0    0    0]
[  72  -24    0    0    0    0]
[  72  -24    0    0    0    0]
[  24    0    0    0   48  -12]
[  24    0    0    0  -24   12]
```

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