R02

Thursday, May 6, 2021 3:01 PM

* Final on Wednesday, May 12 at 8:00 am
* Teaching evaluation
* Review session:

  Monday 3 pm

* sites.google.com/stonybrook.edu/nathanchen/teaching

  ↑ practice final

OH: 6-7pm.

<u>Quiz 11</u>

1) $\lim_{\theta\to\frac{\pi}{2}} \frac{1-\sin\theta}{\csc\theta} \xrightarrow[\text{in}]{\text{plug}} \frac{1-\sin\frac{\pi}{2}}{\csc\frac{\pi}{2}} = \frac{1-1}{\frac{1}{\sin\frac{\pi}{2}}} = \frac{1-1}{\left(\frac{1}{1}\right)} = \frac{0}{1} = 0.$

Cannot apply L'Hôpital's Rule

2) $\lim_{x\to 0} \frac{x+1-e^x}{x^2} \xrightarrow[\text{in}]{\text{plug}} \frac{0+1-e^0}{0^2} = \frac{0+1-1}{0} = \frac{0}{0}$

$\underset{\text{L'H}}{=} \lim_{x\to 0} \frac{1+0-e^x}{2x} \xrightarrow[\text{in}]{\text{plug}} \frac{1-e^0}{2\cdot 0} = \frac{0}{0}$

$\underset{\text{L'H}}{=} \lim_{x\to 0} \frac{-e^x}{2} \underset{\text{plug in}}{=} \frac{-e^0}{2} = -\frac{1}{2}.$

<u>Def</u>. Let $f(x)$ be a function. An anti-derivative of f is another function $F(x)$ such that $F'(x) = f(x)$

<u>Ex</u>. We know that $\frac{d}{dx}(x^2) = 2x$

$F(x) = x^2$ is an antiderivative of $2x$,

$G(x) = x^2+3$ is <u>also</u> an anti-derivative of $2x$.

<u>Terminology</u>: The general antiderivative of $f(x)$ is

$F(x) + C$

<u>Ex</u>. What is the general anti-derivative of

$f(x) = \csc^2 x$?

<u>Recall</u>: $\frac{d}{dx}(\cot x) = -\csc^2 x$

↑ negative sign

$\frac{d}{dx}(-\cot x) = \csc^2 x$

<u>Ans</u>. The general antiderivative is

$F(x) = -\cot x + C$

<u>Ex</u>. $f(x) = x^n \quad (n \neq -1)$

The general anti-derivative is $F(x) = \frac{x^{n+1}}{n+1} + C$.

$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{n+1}\cdot\frac{d}{dx}(x^{n+1})$ ← use power rule

$= \frac{1}{\cancel{n+1}}\cancel{(n+1)}\cdot x^n$

$= x^n$

<u>Ex</u>. $f(x) = x^{-1} = \frac{1}{x}$. $\quad F(x) = \ln|x| + C$

<u>Ex</u>. $f(x) = (x+3)^4$

← $+3$ is a distraction

$F(x) = \frac{(x+3)^5}{5} + C$.

$\frac{d}{dx}\left(\frac{(x+3)^5}{5}\right) = \frac{1}{5}\cdot\frac{d}{dx}\left((x+3)^5\right)$

$= \frac{1}{\cancel{5}}\cdot\cancel{5}\cdot(x+3)^4\cdot\underbrace{\cancel{\frac{d}{dx}(x+3)}}^{1}$

$= (x+3)^4$

<u>Ex</u>. $f''(x) = 4+e^x$ and $f(0) = 2$ $\quad f'(0) = 3$

<u>Q</u>. Find the original function f.

1) Find an anti-der. for $f''(x)$.

$f'(x) = 4x + e^x + C_1$

$f'(0) = 4\cdot 0 + e^0 + C_1 = 1 + C_1 = 3 \Rightarrow C_1 = 2$

So $f'(x) = 4x + e^x + 2$

2) Find anti-der for $f'(x)$.

$f(x) = 4\cdot\frac{x^2}{2} + e^x + 2x + C_2$

$= 2x^2 + e^x + 2x + C_2$

$2 = f(0) = 2\cdot 0^2 + e^0 + 2\cdot 0 + C_2$

$= 1 + C_2 \Rightarrow C_2 = 1$

So $f(x) = 2x^2 + e^x + 2x + 1$

<u>Q</u>. $4x$. Find anti-derivative

Find an anti-der. for x and then multiply by 4.

<u>Claim</u>: $\frac{x^2}{2}$ is an anti-der. of x.

Check $\frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{2}\cdot 2x = x$

<u>Claim</u>: $\underbrace{4\cdot\frac{x^2}{2}}_{2x^2}$ is an anti-der. of $4x$