OH: today 6-7pm

Quiz 7

1. Find $\frac{d}{dx}\left(e^{\arccos x}\right)$ use chain rule

$= e^{\arccos x} \cdot \frac{d}{dx}(\arccos x)$

$= e^{\arccos x} \cdot \frac{(-1)}{\sqrt{1-x^2}}$

2. Use implicit diff to find $\frac{dy}{dx}$:

$y \cdot \sin x + x = x^2 \cdot y.$

Take $\frac{d}{dx}$:

$\underline{\frac{dy}{dx}} \cdot \sin x + y \cdot \cos x + 1 = 2x \cdot y + x^2 \cdot \underline{\frac{dy}{dx}}$

$\frac{dy}{dx} \cdot \sin x - x^2 \cdot \frac{dy}{dx} = 2xy - y \cdot \cos x - 1$

$\|$

$\frac{dy}{dx} \cdot (\sin x - x^2)$

So $\frac{dy}{dx} = \frac{2xy - y \cdot \cos x - 1}{\sin x - x^2}$

Recall $\frac{d}{dx}(\ln x) = \frac{1}{x}.$

$\frac{d}{dx}(\ln g(x)) = \frac{1}{g(x)} \cdot g'(x)$ by chain rule

Ex $\frac{d}{dx}\left(\log_{500} x^3\right)$

$\boxed{\text{Change of base: } \log_b a = \frac{\log_d a}{\log_d b}}$

$= \frac{d}{dx}\left(\frac{\ln x^3}{\ln 500}\right)$

$= \frac{1}{\ln 500} \cdot \boxed{\frac{d}{dx}(\ln x^3)}$

$= \frac{1}{\ln 500} \cdot \frac{1}{x^3} \cdot 3x^2 = \frac{1}{\ln 500} \cdot \frac{3}{x}$

Another way: $\frac{d}{dx}(\ln x^3)$ $\boxed{\log_b a^c = c \cdot \log_b a}$

$= \frac{d}{dx}(3 \cdot \ln x)$

$= 3 \cdot \frac{d}{dx}(\ln x)$

$= \frac{3}{x}$

Logarithm rules

① $\log_b(f(x) \cdot g(x)) = \log_b f(x) + \log_b g(x)$

② $\log_b\left(\frac{f(x)}{g(x)}\right) = \log_b f(x) - \log_b g(x)$

$\|$

$\log_b\left(f(x) \cdot [g(x)]^{-1}\right)$

Ex. $\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right) = \frac{1}{(\ln b) \cdot x}$

Ex $y = (2x^4+1)^{\tan x}$. Use logarithmic diff.

↳ take ln of both sides

↳ simplify using log rules

↳ take $\frac{d}{dx}$ and implicit diff

$\ln y = \ln (2x^4+1)^{\tan x}$

$\ln y = \tan x \cdot \ln(2x^4+1)$

use product rule

Take $\frac{d}{dx}$:

$\frac{1}{y} \cdot \frac{dy}{dx} = \sec^2 x \cdot \ln(2x^4+1) + \tan x \cdot \frac{d}{dx}(\ln(2x^4+1))$

$\frac{1}{y} \cdot \frac{dy}{dx} = \sec^2 x \cdot \ln(2x^4+1) + \tan x \cdot \frac{1}{2x^4+1} \cdot 8x^3$

So $\frac{dy}{dx} = y\left(\sec^2 x \cdot \ln(2x^4+1) + \tan x \cdot \frac{8x^3}{2x^4+1}\right)$

↑

plug in $y = (2x^4+1)^{\tan x}$

Ex $y = \frac{\sqrt{2x+1}}{e^x \cdot \sin^3 x}$ Use log. diff.

$\ln y = \ln\left(\frac{\sqrt{2x+1}}{e^x \cdot \sin^3 x}\right)$ product

$= \ln \sqrt{2x+1} - \ln(e^x \cdot \sin^3 x)$

$= \ln(2x+1)^{1/2} - [\ln e^x + \ln \sin^3 x]$

$\ln y = \frac{1}{2}\ln(2x+1) - \underbrace{\ln e^x}_{x} - \underbrace{\ln \sin^3 x}_{\ln(\sin x)^3 = 3 \cdot \ln(\sin x)}$

Take $\frac{d}{dx}$:

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{\cancel{1}}{\cancel{2}} \cdot \frac{1}{(2x+1)} \cdot \cancel{2} - 1 - 3 \cdot \frac{1}{\sin x} \cdot \cos x$

$\frac{dy}{dx} = y \cdot \left(\frac{1}{2x+1} - 1 - 3 \cdot \cot x\right)$

Plug in $y = \frac{\sqrt{2x+1}}{e^x \cdot \sin^3 x}$:

$\frac{dy}{dx} = \frac{\sqrt{2x+1}}{e^x \cdot \sin^3 x} \cdot \left(\frac{1}{2x+1} - 1 - 3 \cdot \cot x\right)$