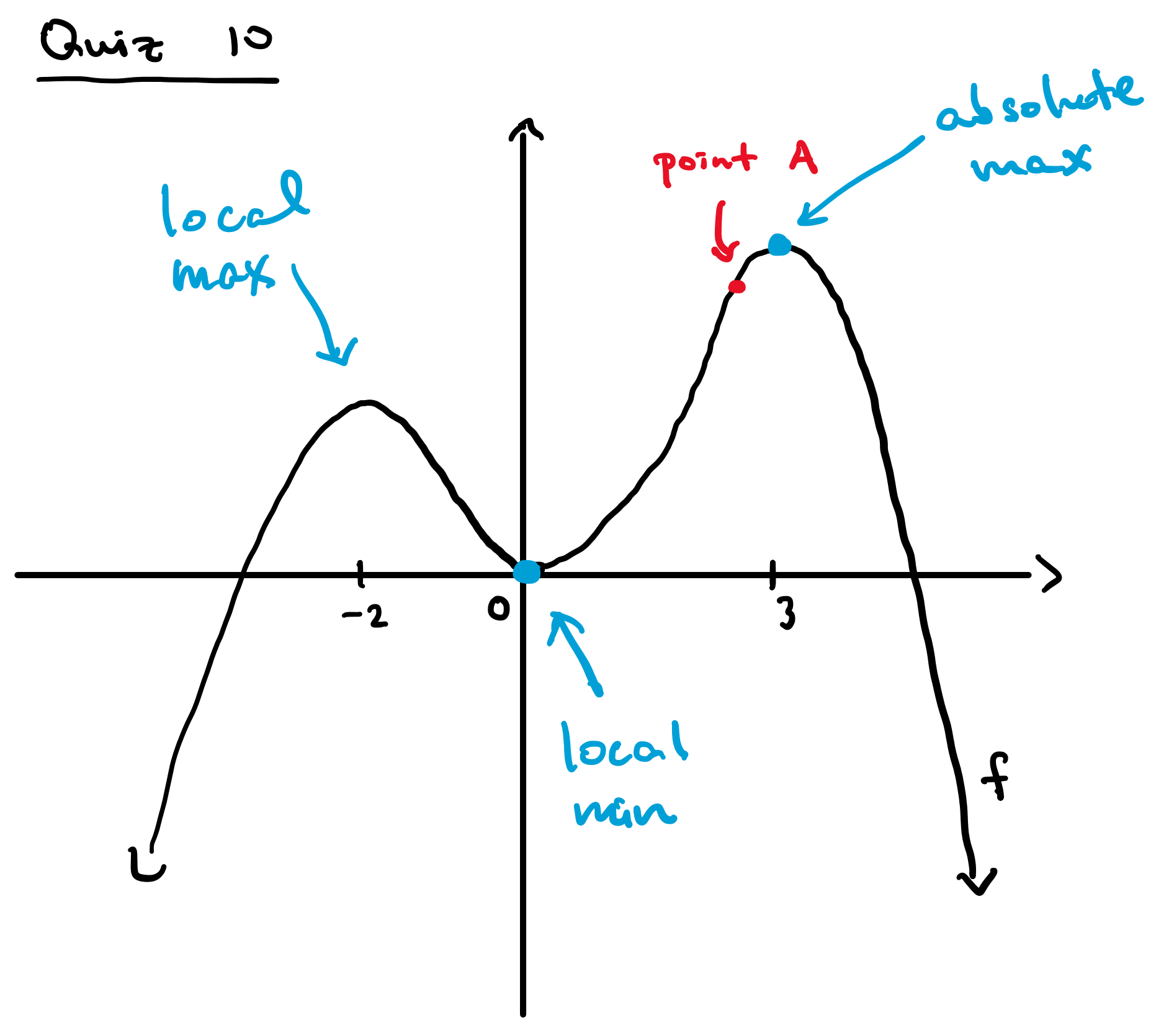
R02

Thursday, April 29, 2021 3:58 PM

* OH: Today 6-7pm Zoom ID: 939 310 5930 (password: math2020).

* Final Exam: May 12 8 - 10:45 am

Quiz 10



(a) $f'(x) > 0 \iff f$ is increasing

$(-\infty, -2) \cup (0, 3)$

(b) $f'(x) < 0 \iff f$ is decreasing

$(-2, 0) \cup (3, \infty)$

(c) $f'(x) = 0 \iff f$ has a horizontal tangent

$x = -2, 0, 3$

NOT same thing as points of inflection

(d) Sign of $f''(x)$ at point A

negative $(-)$ $\iff$ concave down.

(e) Absolute maximum at $x = 3$

(f) no absolute min.

L'Hôpital's Rule:

Suppose $\lim_{x\to a} \frac{f(x)}{g(x)}$ and $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$.

Then $\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$

Ex $\lim_{x\to 0} \frac{1-\cos x}{x} = \frac{1-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$.

So $\lim_{x\to 0} \frac{1-\cos x}{x} \underset{\text{Apply L'H}}{=} \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x)}$

$= \lim_{x\to 0} \frac{\sin x}{1}$

$= \sin 0 = 0$.

Caution: Need to check: $\lim_{x\to a} f(x) = 0$

AND $\lim_{x\to a} g(x) = 0$

Non-example:

Consider $\lim_{x\to 1} \frac{x^2+5}{3x+4} \overset{\text{Try}}{\underset{\text{to apply}}{=}} \lim_{x\to 1} \frac{2x}{3} = \frac{2}{3}$.

However $\Vert$

$\lim_{x\to 1} \frac{x^2+5}{3x+4} \underset{\text{plug in}}{=} \frac{1^2+5}{3\cdot 1+4} = \boxed{\frac{6}{7}}$

Correct answer.

Problem: $\lim_{x\to 1} x^2+5 = 6 \neq 0$ and $\lim_{x\to 1} 3x+4 = 7 \neq 0$.

L'Hôpital's Rule also works for limits which give $\frac{\infty}{\infty}$.

Ex $\lim_{x\to\infty} \frac{3x+5 \to \infty}{2x+4 \to \infty} \underset{\text{L'H}}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(3x+5)}{\frac{d}{dx}(2x+4)}$

$= \lim_{x\to\infty} \frac{3}{2} = \frac{3}{2}$.

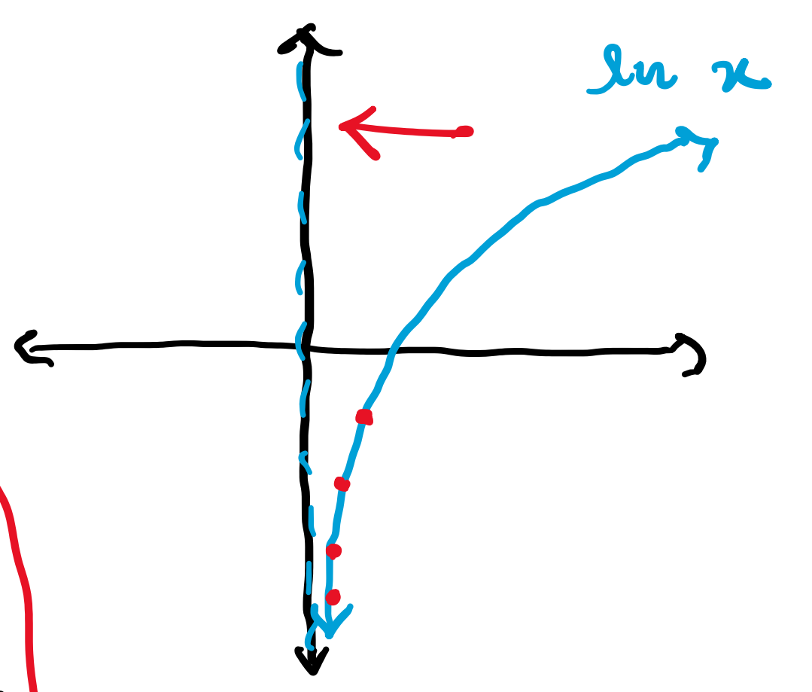
Ex $\lim_{x\to 0^+} \frac{\ln x}{\cot x}$

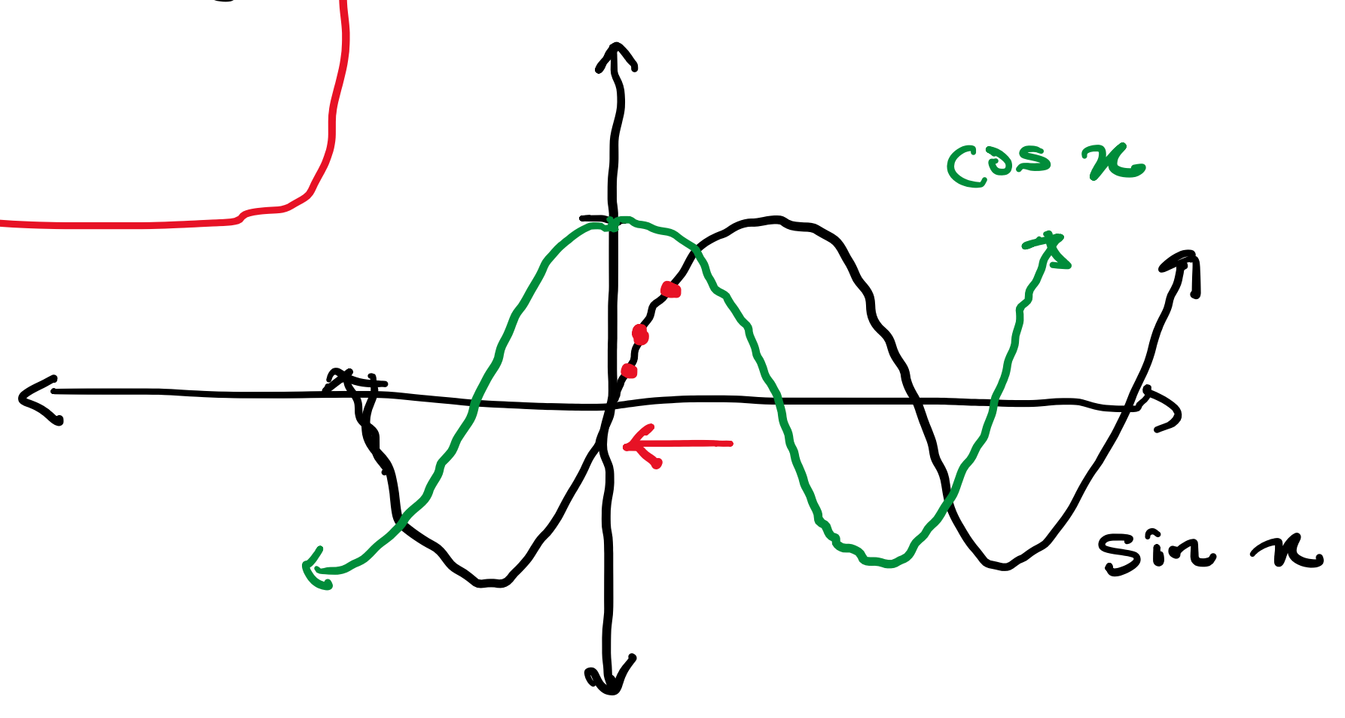
Check: $\lim_{x\to 0^+} \ln x = -\infty$

$\lim_{x\to 0^+} \cot x = \lim_{x\to 0^+} \frac{\cos x \to 1}{\sin x \to 0^+}$

$= +\infty$

This means we can apply L'Hôpital's Rule





Apply L'H

$= \lim_{x\to 0} \frac{\left(\frac{1}{x}\right)}{-\csc^2 x}$

$\csc x = \frac{1}{\sin x}$

$= \lim_{x\to 0} - \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{\sin^2 x}\right)}$

$= \lim_{x\to 0} - \left(\frac{1}{x}\right)\cdot\left(\frac{\sin^2 x}{1}\right)$

$= \lim_{x\to 0} - \frac{\sin^2 x \to 0}{x \to 0}$

$\sin^2 x = (\sin x)^2$

use chain rule

Apply L'H $= \lim_{x\to 0} - \frac{2\cdot(\sin x)\cdot\cos x}{1}$

Try to plug in $= \lim_{x\to 0} - \frac{2\cdot \overset{0}{\cancel{\sin 0}} \cdot \overset{1}{\cancel{\cos 0}}}{1} = \frac{0}{1} = 0$