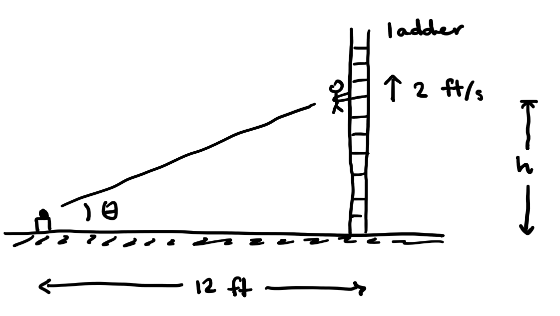
R02

Thursday, April 22, 2021 4:44 PM

Quiz 9

R09



h = height of person climbing ladder

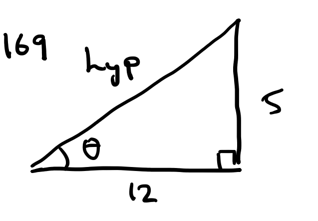
Want to find: $\frac{d\theta}{dt}$ when $h = 5$ ft.

$\tan\theta = \frac{opp}{adj} = \frac{h}{12}$

Take $\frac{d}{dt}$: $\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{12} \cdot \frac{dh}{dt}$

$\frac{d\theta}{dt} = \cos^2\theta \cdot \frac{1}{12} \cdot \frac{dh}{dt}$ $\qquad \frac{dh}{dt} = 2$ ft/s

$hyp^2 = 12^2 + 5^2 = 144 + 25 = 169$

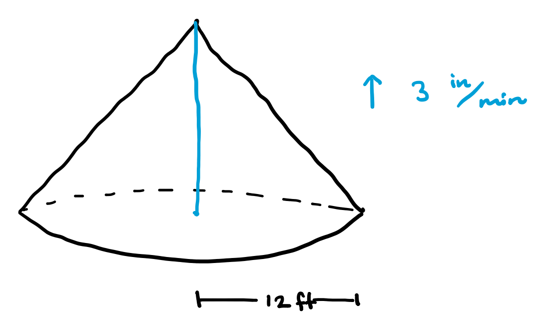


$hyp = \sqrt{169} = 13$

$\cos^2\theta = \left(\frac{adj}{hyp}\right)^2 = \left(\frac{12}{13}\right)^2$

$$\frac{d\theta}{dt} = \frac{1}{12} \cdot \left(\frac{12}{13}\right)^2 \cdot 2 = \frac{12 \cdot 2}{13^2} = \frac{24}{169} \frac{rad}{s}$$

R02



How fast is the volume changing.

Let V = volume of cone.

What is $\frac{dV}{dt}$?

$V = \frac{1}{3} \cdot \pi r^2 \cdot h = \frac{1}{3} \cdot \pi \cdot (12)^2 \cdot h$

Take $\frac{d}{dt}$: $\frac{dV}{dt} = \frac{144\pi}{3} \cdot \frac{dh}{dt}$ $\qquad \frac{dh}{dt} = 3 \frac{in}{min} = \frac{1}{4} \frac{ft}{min}$

$= \frac{144\pi}{3} \cdot \left(\frac{1}{4}\right)$

$= \frac{36\pi}{3}$

$= 12\pi \cdot \frac{ft^3}{min}$

Change in volume does not depend on height.

$V = \frac{1}{3}\pi r^2 \cdot h$ (product rule) $\rightsquigarrow \frac{dV}{dt} = \frac{\pi}{3} \cdot \left(2r \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt}\right)$ ← $\frac{dr}{dt} = 0$

1st and 2nd Derivative test

Def. A function f has a critical point at $x = c$ if

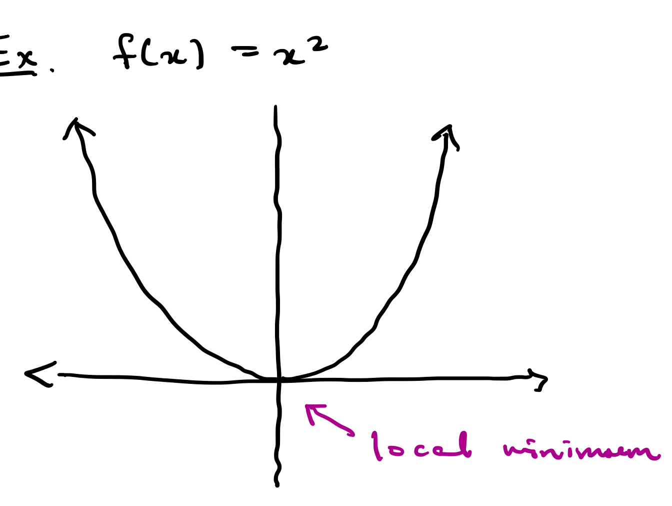
$f'(c) = 0$ or $f'(c) = DNE$. ← graph has a horizontal tangent

1st Derivative test

Suppose f has a critical point at $x = c$.

1. If f' changes sign from + to −, then f has a local maximum at $x = c$.
2. If f' changes sign from − to +, then f has a local minimum at $x = c$.

Ex. $f(x) = x^2$



$f'(x) = 2x \overset{set}{=} 0$

$\Rightarrow x = 0$ critical point

x	−3	0	5
f'(x)	−	0	+

① f is decreasing on $(-\infty, 0)$

② f is increasing on $(0, \infty)$

③ f has a local minimum at $x = 0$.

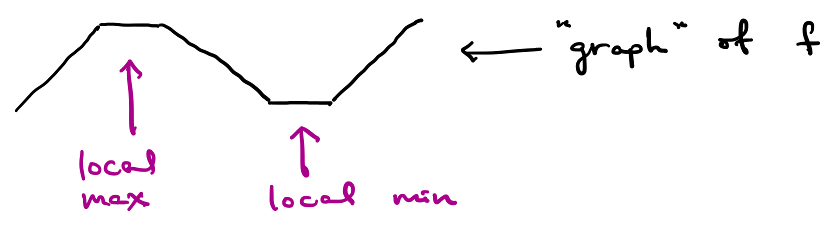
Ex $f(x) = x^3 - 3x^2 - 9x + 1$

$f'(x) = 3x^2 - 6x - 9 \overset{set}{=} 0$

$3(x^2 - 2x - 3) = 0$

$f(x) = 3(x-3)(x+1) = 0 \Rightarrow x = -1, x = 3$ critical points

x	−4	−1	0	3	4
f'(x)	+	0	−	0	+



2nd Derivative Test

Suppose f has a critical point at $x = c$. $f'(c) = 0$

1. If $f''(c) > 0$ (concave up), then f has a local minimum at c.
2. If $f''(c) < 0$, then f has a local maximum at c.

Intuition:

$f'(c) = 0$, $f''(c) > 0$ local min

$f'(c) = 0$, $f''(c) < 0$ local max

Use this to check what we had.

$f(x) = x^3 - 3x^2 - 9x + 1$

$f'(x) = 3x^2 - 6x - 9 \overset{set}{=} 0 \Rightarrow x = -1, x = 3$ critical points

$f''(x) = 6x - 6$

Compute: $f''(-1) = 6 \cdot (-1) - 6 = -12 < 0$ local max

$f''(3) = 6 \cdot 3 - 6 = 12 > 0$ local min

x	0	1	5
f''(x)	−	0	+

sign changes

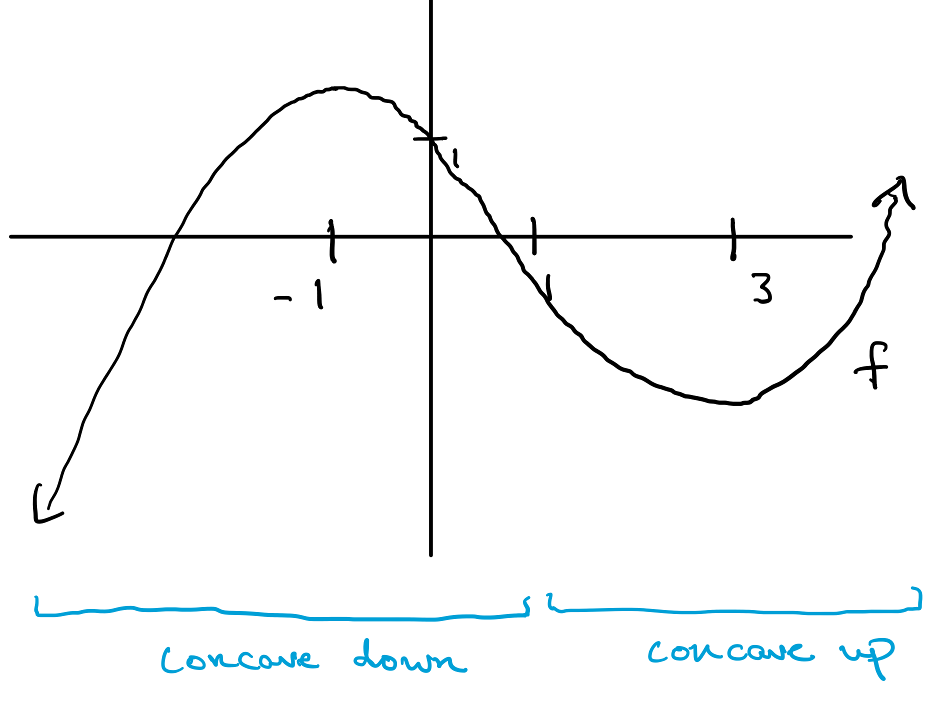
$f''(x) = 6x - 6 = 0$

$x = 1$

① f is concave down on $(-\infty, 1)$

② f is concave up on $(1, \infty)$

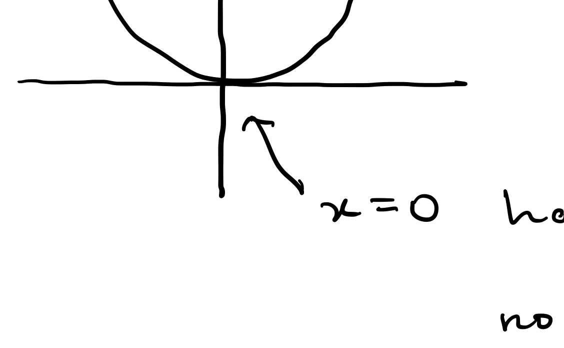
③ f has an inflection point at $x = 1$



$f(x) = x^3 - 3x^2 - 9x + 1$

local max at −1

local min at 3



$x = 0$ have an absolute min.

no absolute max