Thursday, April 15, 2021 4:46 PM

Office hours today 6-7pm

Quiz 8

1. Find derivative $\log_5 \frac{1}{\sqrt{3t+2}}$

$$y = \frac{\ln\left(\frac{1}{\sqrt{3t+2}}\right)}{\ln 5} = \frac{\ln\left((3t+2)^{-\frac{1}{2}}\right)}{\ln 5} = -\frac{\ln(3t+2)}{2\cdot \ln 5}$$

$$\frac{dy}{dt} = -\frac{1}{2\ln 5}\cdot \frac{d}{dt}\left(\ln(3t+2)\right)$$

chain rule

$$= -\frac{1}{2\cdot \ln 5}\cdot \frac{1}{(3t+2)}\cdot \frac{d}{dt}(3t+2)$$

$$= -\frac{3}{2\cdot \ln 5\cdot (3t+2)}$$

2. Use logarithmic diff to find $\frac{dy}{dx}$ for

$$y = (e^x+1)^6\cdot (x^2+3)^7$$

Take ln of both sides

$$\ln y = \ln\left[(e^x+1)^6\cdot (x^2+3)^7\right]$$

$$= \ln(e^x+1)^6 + \ln(x^2+3)^7$$

$$= 6\cdot \ln(e^x+1) + 7\cdot \ln(x^2+3)$$

Take $\frac{d}{dx}$:

$$\frac{1}{y}\cdot\frac{dy}{dx} = 6\cdot \frac{1}{e^x+1}\cdot \underbrace{\frac{d}{dx}(e^x+1)}_{e^x} + 7\cdot\frac{1}{x^2+3}\cdot \underbrace{\frac{d}{dx}(x^2+3)}_{2x}$$

$$= \frac{6e^x}{e^x+1} + \frac{14x}{x^2+3}$$

$$\frac{dy}{dx} = y\cdot\left(\frac{6e^x}{e^x+1} + \frac{14x}{x^2+3}\right)$$

$$\frac{dy}{dx} = (e^x+1)^6\cdot(x^2+3)^7\cdot\left(\frac{6e^x}{e^x+1} + \frac{14x}{x^2+3}\right)$$

Related Rates

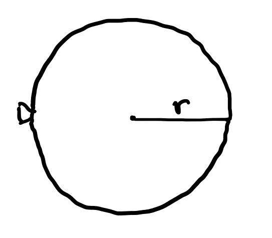
1. Draw a picture of the problem and label

2. Identify what you are solving for

3. Find an equation involving all variables (simplify if possible)

4. Take the derivative (usually $\frac{d}{dt}$) where t = time

Ex: A spherical balloon is being filled with air at a constant rate of 2 cm³/sec. How fast is the radius increasing when radius = 3 cm.



V = volume of balloon

$$\boxed{\frac{dV}{dt} = 2 \ \text{cm}^3/\text{sec}}$$

r = radius of balloon.

We want $\frac{dr}{dt}$ when $\boxed{r = 3 \text{ cm.}}$

$V = \frac{4\pi}{3}\cdot r^3$ volume of sphere.

$r^3 = (r(t))^3$

$\frac{d}{dt}(r^3) = 3\cdot (r(t))^2\cdot \frac{dr}{dt}$

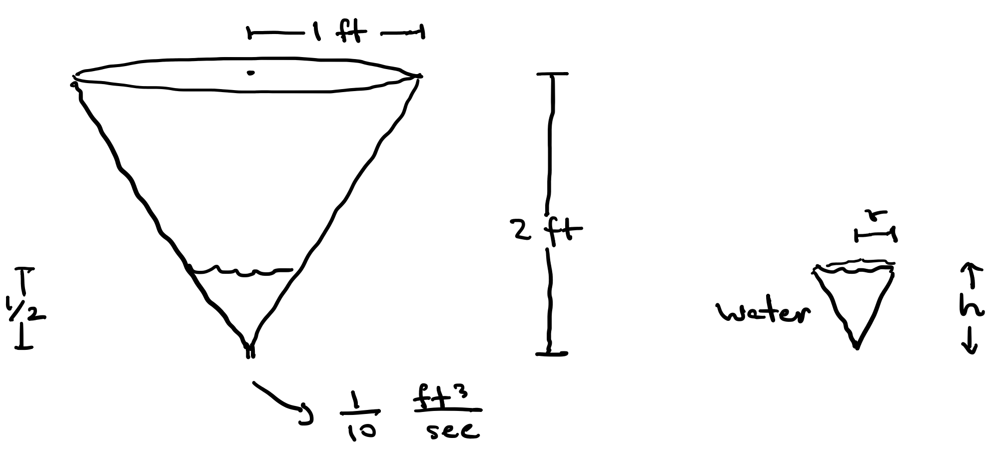
Take $\frac{d}{dt}$: $\frac{dV}{dt} = \frac{4\pi}{3}\cdot \underbrace{3r^2\cdot\frac{dr}{dt}}_{\frac{d}{dt}(r^3)}$

$$2 = \frac{4\pi}{\cancel{3}}\cdot \cancel{3}\cdot(3)^2\cdot\frac{dr}{dt}$$

So $\frac{dr}{dt} = \frac{2}{4\pi\cdot 3^2} = \frac{1}{18\pi} \frac{\text{cm}}{\text{sec}}.$

r is in cm
t is in sec

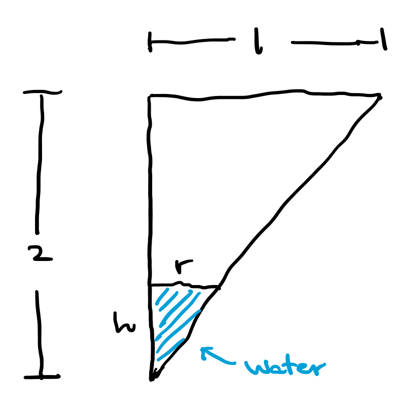
Ex. Water is draining from the bottom of a cone at a rate of $\frac{1}{10}\frac{ft^3}{sec}$. Cone is 2ft tall and radius of the top of funnel is 1 ft. How fast is the height of water changing when height of water is $\frac{1}{2}$ ft?



We want to find $\boxed{\frac{dh}{dt}}$ when $\boxed{h = \frac{1}{2}}$

Given $\frac{dV}{dt} = -\frac{1}{10}\frac{ft^3}{sec}$ where V = volume of water

$V = \frac{1}{3}\cdot\pi r^2\cdot h$ ← try to simplify.



$\frac{r}{h} = \frac{1}{2}$ get this from similar triangles

Solve for $r = \frac{1}{2}h$

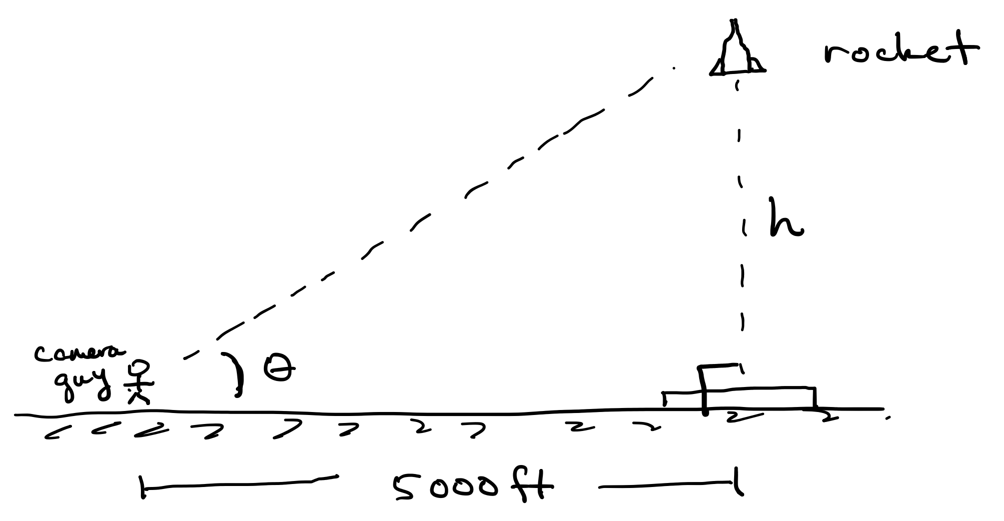
$$V = \frac{1}{3}\cdot\pi\cdot\left(\frac{1}{2}h\right)^2\cdot h = \frac{\pi}{12}\cdot h^3$$

Take $\frac{d}{dt}$: $\frac{dV}{dt} = \frac{\pi}{\cancel{12}_4}\cdot\cancel{3}\cdot h^2\cdot\frac{dh}{dt}$

$$-\frac{1}{10} = \frac{\pi}{4}\cdot\left(\frac{1}{2}\right)^2\cdot\frac{dh}{dt}$$

$$-\frac{1}{10} = \frac{\pi}{16}\cdot\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{16}{10\pi} = -\frac{8}{5\pi}\ \frac{ft}{sec}$$

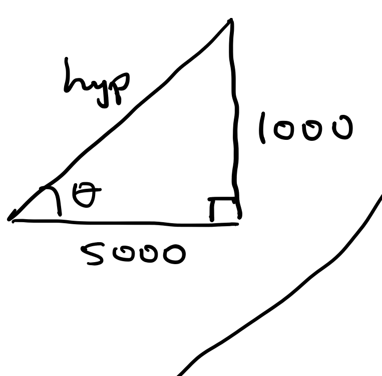
Ex A rocket is being launched vertically. A camera is positioned 5000 ft from launchpad. When height is 1000 ft, the velocity is 600 ft/sec. Find rate of change of camera's angle.



We want $\boxed{\frac{d\theta}{dt}}$ when $h = 1000$ ft.

Equation: $\tan\theta = \frac{h}{5000}$

Take $\frac{d}{dt}$: $\boxed{\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{1}{5000}\cdot\frac{dh}{dt}}$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} = \frac{\sqrt{26000000}}{5000}$$

$$(hyp)^2 = 5000^2 + 1000^2$$

$$= 26000000$$

$$hyp = \sqrt{26000000}$$

$$\left(\frac{\sqrt{26000000}}{5000}\right)^2\cdot\frac{d\theta}{dt} = \frac{1}{5000}\cdot 600$$

$$\Rightarrow \frac{26}{25}\cdot\frac{d\theta}{dt} = \frac{6}{50} \Rightarrow \frac{d\theta}{dt} = \frac{\cancel{6}^{3}}{\cancel{50}_{2}}\cdot\frac{\cancel{25}}{26} = \frac{3}{26}\ \frac{rad}{sec}.$$

$\frac{d\theta}{dt}$ ← θ is in radians

↖ t is in sec.

So $\frac{d\theta}{dt}$ is in $\frac{rad}{sec}$.