R02

Thursday, April 1, 2021 4:43 PM

Note: no office hours today

Last week's quiz

1. Position $s(t) = \cos(t^2) + t^2$

(a) Velocity $v(t) = s'(t)$

$$= \underbrace{-\sin(t^2)}_{\textcircled{1}} \cdot \underbrace{2t}_{\textcircled{2}} + 2t$$

product rule

(b) Acceleration $a(t) = v'(t)$

$$= \underbrace{-\cos(t^2)\cdot 2t}_{\textcircled{1}'} \cdot 2t + (-\sin(t^2))\cdot 2 + 2$$

(c) $v(0) = -\sin(0)\cdot 2\cdot 0 + 2\cdot 0 = 0$

(d) $a(0) = -\cos\cancel{(0)\cdot 2\cdot 0\cdot 2\cdot 0} - \cancel{\sin(0)\cdot 2} + 2 = 2.$

2. A student computes $\frac{d}{dx}\left(\frac{x^2}{x^2+1}\right) \overset{?}{=} \frac{2x}{x+1}$.

Quotient rule $\left(\frac{t}{b}\right)' = \frac{b\cdot t' - t\cdot b'}{b^2}$

$$\frac{d}{dx}\left(\frac{x^2}{x^2+1}\right) = \frac{(x^2+1)\cdot 2x - x^2\cdot(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3}}{(x^2+1)^2}$$

$$= \frac{2x}{(x^2+1)^2} \neq \frac{2x}{x+1}$$

Ex Assume y is defined by

$$x^3\cdot\sin y + y = 4x + 3$$

Use implicit diff. to find $\frac{dy}{dx}$.

① Take $\frac{d}{dx}$ of both sides

$$\frac{d}{dx}(\underbrace{x^3}\cdot\underbrace{\sin y} + y) = 3x^2\cdot\sin y + x^3\cdot\cos y\cdot\frac{dy}{dx} + \frac{dy}{dx}$$

Use product rule

view as a function of x

$$\frac{d}{dx}(4x+3) = 4$$

$$3x^2\cdot\sin y + x^3\cdot\cos y\cdot\frac{dy}{dx} + \frac{dy}{dx} = 4$$

② Solve for $\frac{dy}{dx}$:

$$x^3\cdot\cos y\cdot\frac{dy}{dx} + 1\cdot\frac{dy}{dx} = 4 - 3x^2\cdot\sin y$$

$$\frac{dy}{dx}(x^3\cdot\cos y + 1) = 4 - 3x^2\cdot\sin y$$

$$\frac{dy}{dx} = \frac{4 - 3x^2\cdot\sin y}{x^3\cdot\cos y + 1}$$

Inverse Trig

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Ex $y\cdot\sqrt{x+4} = xy + 8$ Use implicit diff to find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides

$$\frac{d}{dx}\left(y\cdot\sqrt{x+4}\right) = \frac{d}{dx}\left(\underbrace{y}\cdot\underbrace{(x+4)^{1/2}}\right)$$

$$= \frac{dy}{dx}\cdot(x+4)^{1/2} + y\cdot\underbrace{\frac{d}{dx}\left((x+4)^{1/2}\right)}_{\frac{1}{2}(x+4)^{-\frac{1}{2}}\cdot 1}$$

$$\frac{1}{2}(x+4)^{-\frac{1}{2}}\cdot 1 = \frac{1}{2\sqrt{x+4}}$$

$$\frac{d}{dx}(x\underline{y} + 8) = 1\cdot y + x\cdot\frac{dy}{dx} + 0$$

$$\frac{dy}{dx}\cdot(x+4)^{\frac{1}{2}} + y\cdot\frac{1}{2}(x+4)^{-\frac{1}{2}} = y + x\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\cdot(x+4)^{\frac{1}{2}} - x\cdot\frac{dy}{dx} = y - \frac{1}{2}y\cdot(x+4)^{-\frac{1}{2}}$$

$$\frac{dy}{dx}\left[(x+4)^{\frac{1}{2}} - x\right] = y - \frac{1}{2}y\cdot(x+4)^{-\frac{1}{2}}$$

$$(x+4)^{-\frac{1}{2}} = \frac{1}{(x+4)^{\frac{1}{2}}} = \frac{1}{\sqrt{x+4}}$$

$$\boxed{\frac{dy}{dx} = \frac{y - \frac{1}{2}y\cdot(x+4)^{-\frac{1}{2}}}{(x+4)^{\frac{1}{2}} - x}}$$

↑ substitute y in terms of x (if possible)