R09

Thursday, March 4, 2021 3:01 PM

OH: 6-7pm

Quiz 3.

1. $f(x) = \begin{cases} c \cdot e^x & x < 0, \\ \frac{1}{2}x + 2 & x \geq 0. \end{cases}$ Find c such that f is continuous.

$f(0) = \frac{1}{2}(0) + 2 = 2$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{1}{2}x + 2 = \frac{1}{2} \cdot (0) + 2 = 2.$

↑ need to agree!

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} c \cdot e^x = c \cdot e^0 = c.$

$e^{1+0} = e^1$

$e^1 \cdot e^0 = e^1$

$e^0 = 1$

$c = 2$

2. $\lim_{y \to 16} \frac{4 - \sqrt{y}}{16y - y^2}$ ← multiply by conjugate

$= \lim_{y \to 16} \frac{4 - \sqrt{y}}{16y - y^2} \cdot \frac{(4 + \sqrt{y})}{(4 + \sqrt{y})}$

$= \lim_{y \to 16} \frac{4^2 - (\sqrt{y})^2}{y(16 - y) \cdot (4 + \sqrt{y})}$

$= \lim_{y \to 16} \frac{16 - y}{y \cdot (16 - y) \cdot (4 + \sqrt{y})}$

$= \lim_{y \to 16} \frac{1}{y \cdot (4 + \sqrt{y})}$

$= \frac{1}{16 \cdot (4 + \sqrt{16})} = \frac{1}{16 \cdot 8} = \frac{1}{128}$

Limits of rational functions at $\pm\infty$.

$f(x) = \frac{p(x)}{q(x)}$ ← both polynomials

How to evaluate $\lim_{x \to \infty} \frac{p(x)}{q(x)}$ or $\lim_{x \to -\infty} \frac{p(x)}{q(x)}$.

Consider highest order terms in p, q.

Ex. $p(x) = 3x^3 - 5x^2 + 2$.

↑ highest order term.

Suppose $\lim_{x \to \infty} \frac{p(x)}{q(x)} = \lim_{x \to \infty} \frac{ax^k + (\text{smaller terms})}{b \cdot x^l + (\text{smaller terms})}$ ← higher order terms

$= \lim_{x \to \infty} \frac{a \cdot x^k}{b \cdot x^l}$

1) If $k = l \Rightarrow x^k$ cancels with x^l

So $\lim_{x \to \infty} \frac{a \cdot x^k}{b \cdot x^l} = \frac{a}{b}$. $\lim_{x \to -\infty} \frac{a \cdot x^k}{b \cdot x^l} = \frac{a}{b}$.

2) If $k > l$.

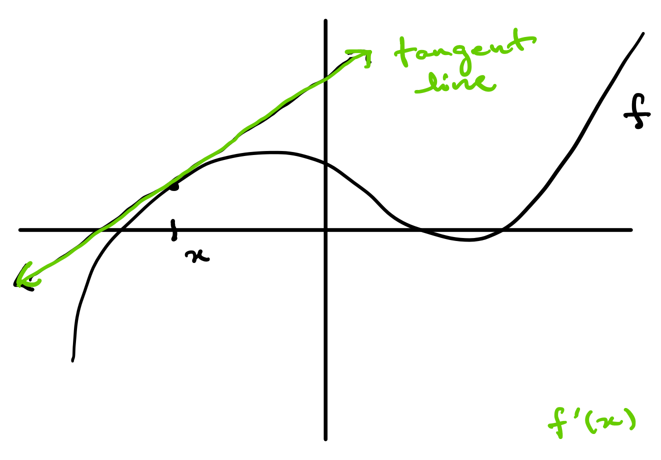
$\lim_{x \to \infty} \frac{3x^2 + 4x}{x + 2} = \lim_{x \to \infty} \frac{3x^2}{x} = \lim_{x \to \infty} 3x = \infty$

$\lim_{x \to -\infty}$ — " — = — " — $= \lim_{x \to -\infty} 3 \cdot x = -\infty$

3) If $k < l$.

$\lim_{x \to -\infty} \frac{3x^2 + 2x}{4x^3 - 5x + 7} = \lim_{x \to -\infty} \frac{3x^2}{4x^3} = \lim_{x \to -\infty} \frac{3}{4 \cdot x} = 0$

Def. Given a function f, the derivative of f is another function f' whose y-values give the slope of the tangent line to f at x.

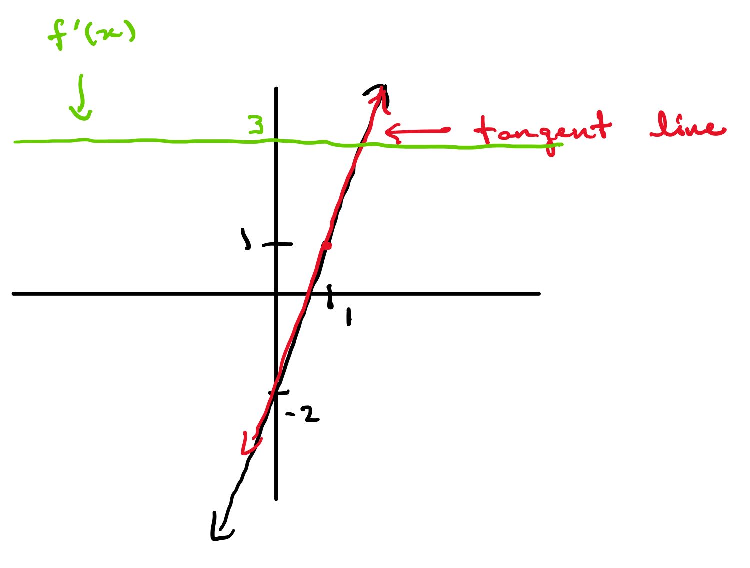


$f'(x)$ = slope of tangent line at x

Ex $f(x) = 3x - 2$

$f'(x) = ?$

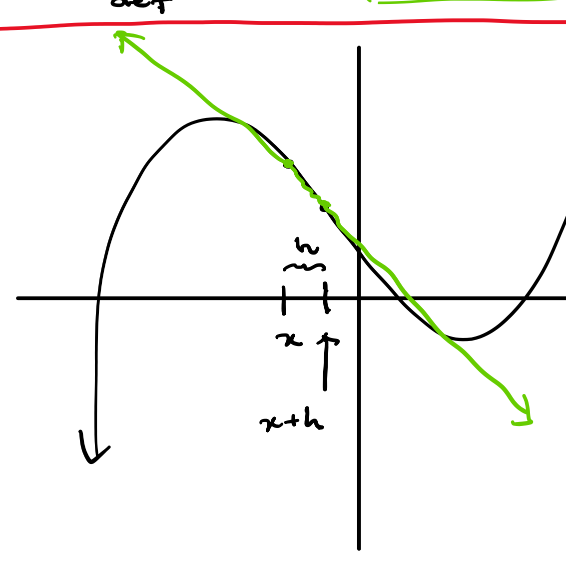
So $f'(x) = 3$



Limit definition

Let f be continuous at x-values

Then $f'(x) \underset{def}{=} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ← slope of secant line



Ex Find derivative of $f(x) = x^2 - 5$ (using limit def)

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 5 - (x^2 - 5)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$

factored

$= \lim_{h \to 0} \frac{h(2x + h)}{h}$

$= \lim_{h \to 0} 2x + h = 2x$

OH:

$f(x) = 4x^2 - 5$. Find eqn of tangent line at $x = 1$.

$f'(x) = 8x \Rightarrow f'(1) = 8$

$f(1) = 4 \cdot 1 - 5 = -1$

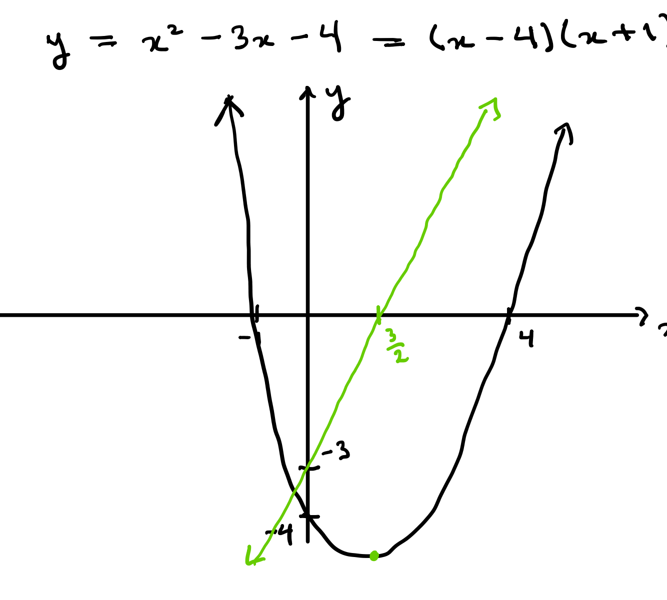
Line w/ slope = 8, point $(1, -1)$ ≈ (x_1, y_1)

$y - y_1 = m \cdot (x - x_1)$

↑ slope

$y + 1 = 8 \cdot (x - 1)$

Ex $y = x^2 - 3x - 4 = (x - 4)(x + 1)$



$f(x) = x^2 - 3x - 4$

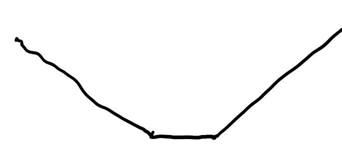
$f'(x) = 2x - 3$

1st Derivative Test:

Set $f'(x) = 2x - 3 = 0 \Rightarrow x = \frac{3}{2}$. ← critical point

x		$\frac{3}{2}$	
$f'(x)$	−	0	+

So f is decreasing (−); f is increasing (+)



sketch of f