R02

Thursday, March 25, 2021 4:31 PM

Quotient Rule $f(x) = \frac{t(x)}{b(x)}$

$$\boxed{f'(x) = \frac{b \cdot t' - t \cdot b'}{b^2}}$$

Position function $s(t)$

velocity: $v(t) = s'(t)$

acceleration: $a(t) = v'(t)$

Warm-up. A car has position given by $s(t) = t \cdot e^{t^3}$

(a) How far has the car traveled from time $t = 1$ to time $t = 2$?

(b) What is the velocity of car at time $t = 1$?

(c) What is the acceleration at $t = 1$?

(a) Find $s(2) - s(1) = 2 \cdot e^{2^3} - e^{1^3} = 2 \cdot e^8 - e.$ $2^3 = 2 \cdot 2 \cdot 2 = 8.$

$s(t) = t \cdot e^{t^3}$ plug in $t = 2$

$s(2) = 2 \cdot e^{(2^3)} = 2 \cdot e^8$ and $s(1) = 1 \cdot e^{(1^3)} = e$

(b) Find $v(1)$. $s(t) = \underbrace{t}_{①} \cdot \underbrace{e^{t^3}}_{②}$

$s'(t) = v(t) = ①' \cdot ② + ① \cdot ②'$

$v(t) = \underbrace{1}_{①'} \cdot \underbrace{e^{t^3}}_{②} + \underbrace{t}_{①} \cdot \underbrace{e^{t^3} \cdot 3 \cdot t^2}_{②'}$

$e^{t^3} = f(g(t))$ where $f(t) = e^t$ ⟶ $f'(t) = e^t$

$g(t) = t^3$ ⟶ $g'(t) = 3t^2$

Chain rule

derivative: $f'(g(t)) \cdot g'(t)$

$e^{g(t)} \cdot g'(t)$

$= e^{t^3} \cdot 3t^2$

$v(t) = e^{t^3} + \underbrace{3 \cdot t^3}_{①} \cdot \underbrace{e^{t^3}}_{②}$ use product rule

$v(1) = e^1 + 3 \cdot 1 \cdot e^1$

$= 4e.$

(c) $a(t) = e^{t^3} \cdot 3t^2 + \underbrace{9 \cdot t^2}_{①'} \cdot \underbrace{e^{t^3}}_{②} + \underbrace{3 \cdot t^3}_{①} \cdot \underbrace{e^{t^3} \cdot 3t^2}_{②'}$

$a(1) = e^1 \cdot 3 + 9 \cdot e^1 + 3 \cdot e^1 \cdot 3$

$= 3e + 9e + 9e$

$= 21e$

Implicit Differentiation

The idea is to rewrite your function in terms of functions whose derivatives are known.

Ex. $y = x^x$

① Take ln of both sides $\frac{d}{dx}(\ln x) = \frac{1}{x}.$

$\ln y = \ln(x^x)$

$\ln y = x \cdot \ln x$

② Take $\frac{d}{dx}$ of both sides

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \cdot \ln x)$

use chain rule Use product rule

$\frac{1}{y} \cdot \underbrace{\frac{dy}{dx}}_{\text{what we want}}$ $1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + 1$

③ Solve: $\frac{dy}{dx} = y \cdot (\ln x + 1)$

plug in $y = x^x$

$\frac{dy}{dx} = x^x \cdot (\ln x + 1)$