OH : 6-7 pm. Midterm next week Wed 4:25 pm

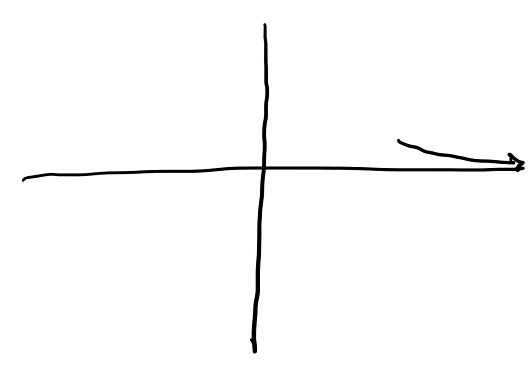
sites.google.com/stonybrook.edu/nathanchen/teaching

Quiz for R09

1. $g(x) = \dfrac{\overbrace{6x^2 + 3x^2}^{9x^2} - 4x}{3x^3 - 2x + 1}$

a. Find $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \dfrac{9x^2}{3x^3} = \lim_{x\to\infty} \dfrac{3}{x} = 0.$

b.



III.

2. $f(x) = 2x^2 - 4x$

a. Use def. of derivative:

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$f(x) = 2x^2 - 4x$

$f(x+h) = 2\cdot(x+h)^2 - 4\cdot(x+h)$

$= \lim_{h\to 0} \dfrac{2\cdot(x+h)^2 - 4\cdot(x+h) - [2x^2 - 4x]}{h}$

$= \lim_{h\to 0} \dfrac{2\cdot(x^2 + 2xh + h^2) - 4x - 4h - 2x^2 + 4x}{h}$

$= \lim_{h\to 0} \dfrac{2x^2 + 4xh + 2h^2 - 4x - 4h - 2x^2 + 4x}{h}$

$= \lim_{h\to 0} \dfrac{4xh + 2h^2 - 4h}{h} = \lim_{h\to 0} \dfrac{h(4x + 2h - 4)}{h}$

$= 4x + 2(0) - 4 = 4x - 4$

b. For what x is the slope of tangent $= 6$?

Set $6 = f'(x) = 4x - 4 \Rightarrow 6 = 4x - 4 \Rightarrow 4x = 10$

$\Rightarrow x = \frac{10}{4} = \frac{5}{2}$

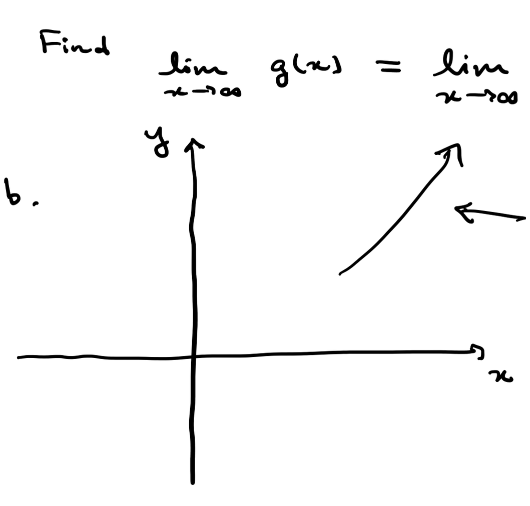
$f'(\frac{5}{2}) = 6$

Quiz for R02

1. a. $g(x) = \dfrac{6x^3 + 3x^2 - 4x}{3x^2 - 2x + 1}$

Find $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \dfrac{6x^3}{3x^2} = \lim_{x\to\infty} 2x = \infty$

b.



II.

2. $f(x) = 2x^2 - 6x$

a. Use def of derivative:

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$f(x) = 2x^2 - 6x$

$f(x+h) = 2\cdot(x+h)^2 - 6\cdot(x+h)$

$= \lim_{h\to 0} \dfrac{2\cdot(x+h)^2 - 6\cdot(x+h) - (2x^2 - 6x)}{h}$

$= \lim_{h\to 0} \dfrac{2(x^2 + 2xh + h^2) - 6x - 6h - 2x^2 + 6x}{h}$

$= \lim_{h\to 0} \dfrac{2x^2 + 4xh + 2h^2 - 6x - 6h - 2x^2 + 6x}{h}$

$= \lim_{h\to 0} \dfrac{h(4x + 2h - 6)}{h}$

$= \lim_{h\to 0} 4x + 2h - 6$

$= 4x + 2(0) - 6 = 4x - 6$

b. For what x is slope of tangent line $= 2$?

Set $2 = f'(x) = 4x - 6$

$2 = 4x - 6 \Rightarrow 4x = 8$

$\Rightarrow x = 2$

Warm-up:

$f(x)$	$f'(x)$	
x^3	$3x^2$	
x^n $(n \neq -1)$	$n \cdot x^{n-1}$	(Power rule)
$x^{-1} = \frac{1}{x}$	$-1 \cdot x^{-2}$	
e^x	e^x	
$\ln x$	$\frac{1}{x}$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$\cot x$	$-\csc^2 x$	
$\sec x$	$\tan x \cdot \sec x$	
$\csc x$	$-\cot x \cdot \csc x$	

Ex. $f(x) = 2x^3 - 6x^2 + 3$

$f'(x) = 2\cdot 3x^2 - 6\cdot 2x$

$= 6x^2 - 12x$

Product Rule: If $f(x) = g(x)\cdot h(x)$, then

$f'(x) = g'(x)\cdot h(x) + g(x)\cdot h'(x)$

Ex. $f(x) = \underbrace{5x^3}_{g(x)} \cdot \underbrace{(\sin x + 2)}_{h(x)}$ $\approx (\sin x + 2)\, 5x^3$

$f'(x) = g'(x)\cdot h(x) + g(x)\cdot h'(x)$

$= 15x^2\cdot(\sin x + 2) + 5x^3 \cdot \cos x$

Quotient Rule $f(x) = \dfrac{g(x)}{h(x)}$

order matters

Then $f'(x) = \dfrac{h(x)\cdot g'(x) - g(x)\cdot h'(x)}{[h(x)]^2}$

Ex. $f(x) = 3x\cdot(18x^4 + \frac{13}{x+1})$

Chain rule $f(x) = g(h(x))$

Then $f'(x) = g'(h(x)) \cdot h'(x)$.

Ex $f(x) = 5\cdot(\underbrace{\tan x - 3}_{h(x)})^4$ $\quad h'(x) = \sec^2 x$

$g(x) = 5x^4 \Rightarrow g'(x) = 20\cdot x^3$

$f'(x) = 20\,(h(x))^3 \cdot h'(x)$

$= 20\,(\tan x - 3)^3 \cdot \sec^2 x$

1st Derivative Test

$f'(x) \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \Rightarrow \begin{cases} f \text{ is increasing} \\ f \text{ has a horizontal tangent} \\ f \text{ is decreasing} \end{cases}$

Ex $f(x) = 2x^3 - 6x^2 + 3$

$f'(x) = 6x^2 - 12x = 0$ 1st step.

$6x^2 - 12x = 6x(x-2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

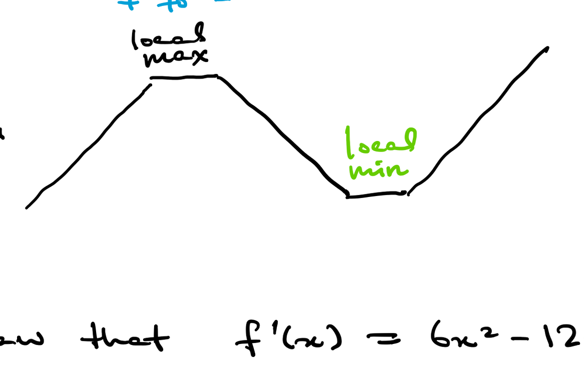
critical points of f

plug into $f'(x)$ to get signs

(−1) (1)

x		0		2	
$f'(x)$	+	0	−	0	+

going from + to −



We saw that $f'(x) = 6x^2 - 12x$

Set $f''(x) = 12x - 12 = 0$

Solve: $x = 1$

x		1	
$f''(x)$	−	0	+

f is concave down — f is concave up