R02 2/4/2021

Thursday, February 4, 2021 4:44 PM

OH: Th 6-7pm. ID: 939 310 5930
password: math2020

MLC website: math.stonybrook.edu/mlc
→ "Center Hours"
my hours: Tu 5-7 pm MLC hours

Quizzes: last ~15 min of each recitation.
upload a picture/pdf onto Blackboard.
↳ assignments

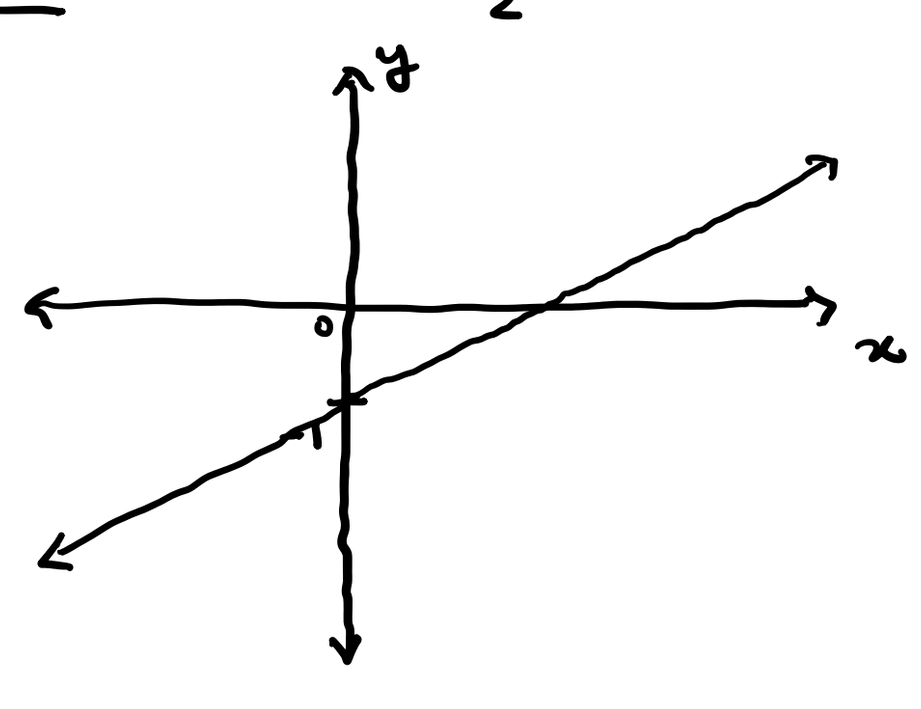
Textbook: Calculus volume 1, OpenStax

my website: sites.google.com/stonybrook.edu/nathanchen/teaching

Review:

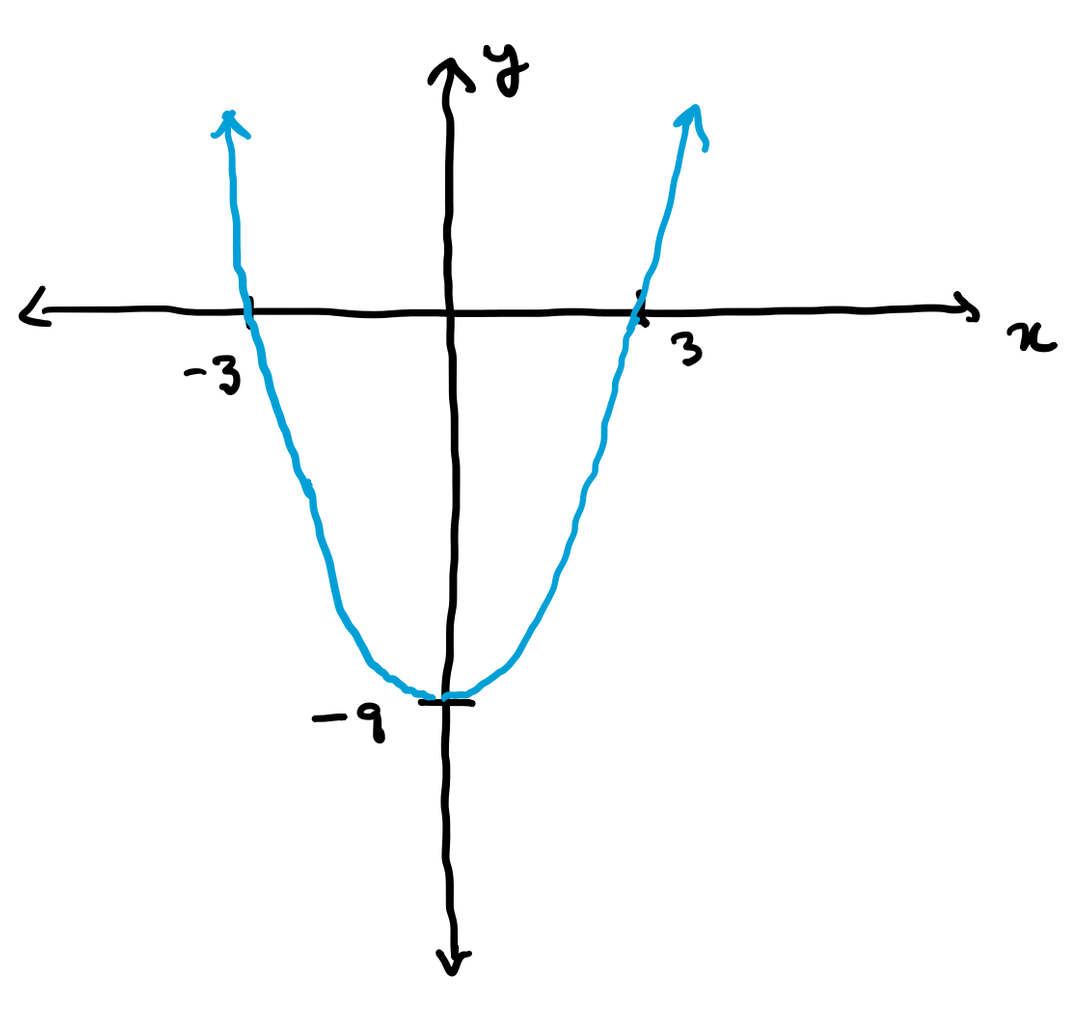
Function – input and output
↑ x ↑ y or $f(x)$

Ex. $f(x) = \frac{1}{2}x - 1$ $f(x) = mx + b$ (↑ slope, ↑ y-intercept)



Ex $f(x) = x^2 - 9$ difference of squares
$= (x+3)(x-3)$

Zeros of f: $x = -3$ and $x = 3$



Ex. $f(x) = -x^2 - 2x + 3$
$= (-x + 1)(x + 3)$

cross terms are $-3x + x = -2x$ ✓

$(-x+1)(x+3) = (-x)\cdot x + (-x)\cdot 3 + x + 3$

check that this adds up.

Zeros of f: $x = 1$ and $x = -3$.

y-intercept $= 3$

Trig functions: examples are $f(x) = \sin x$
$g(x) = \cos x$

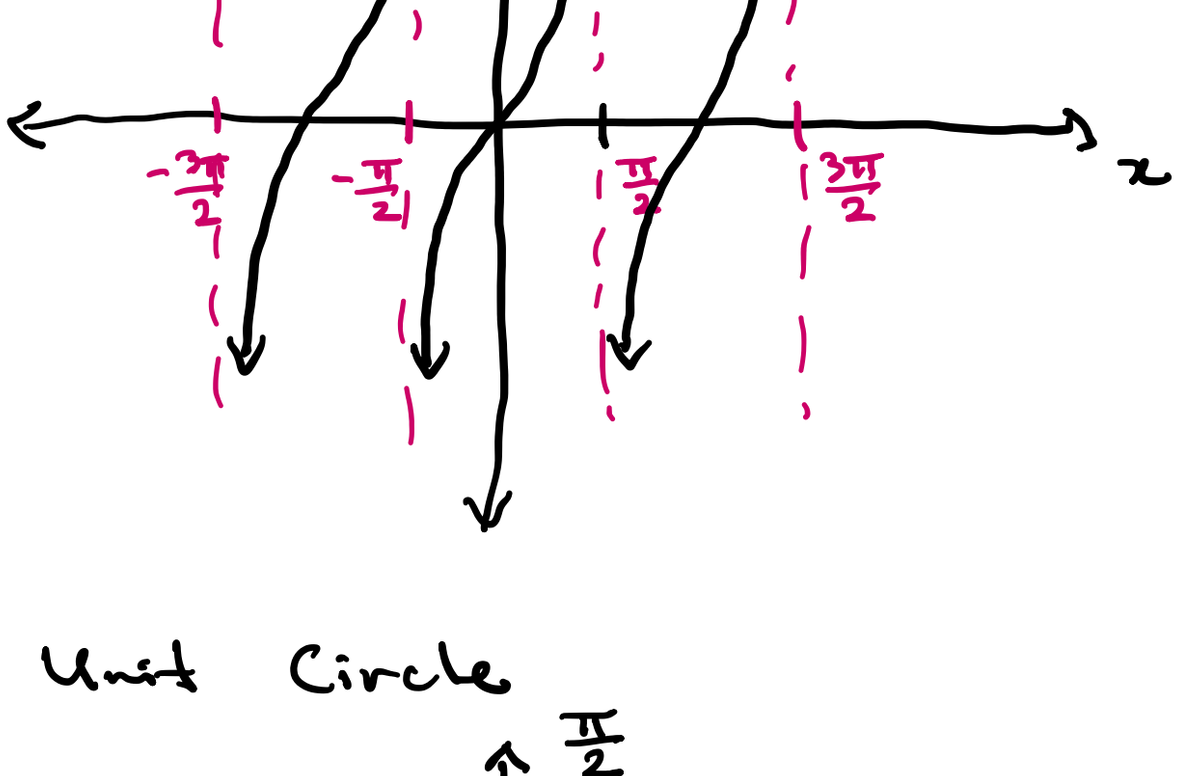
$\tan x = \frac{\sin x}{\cos x}$

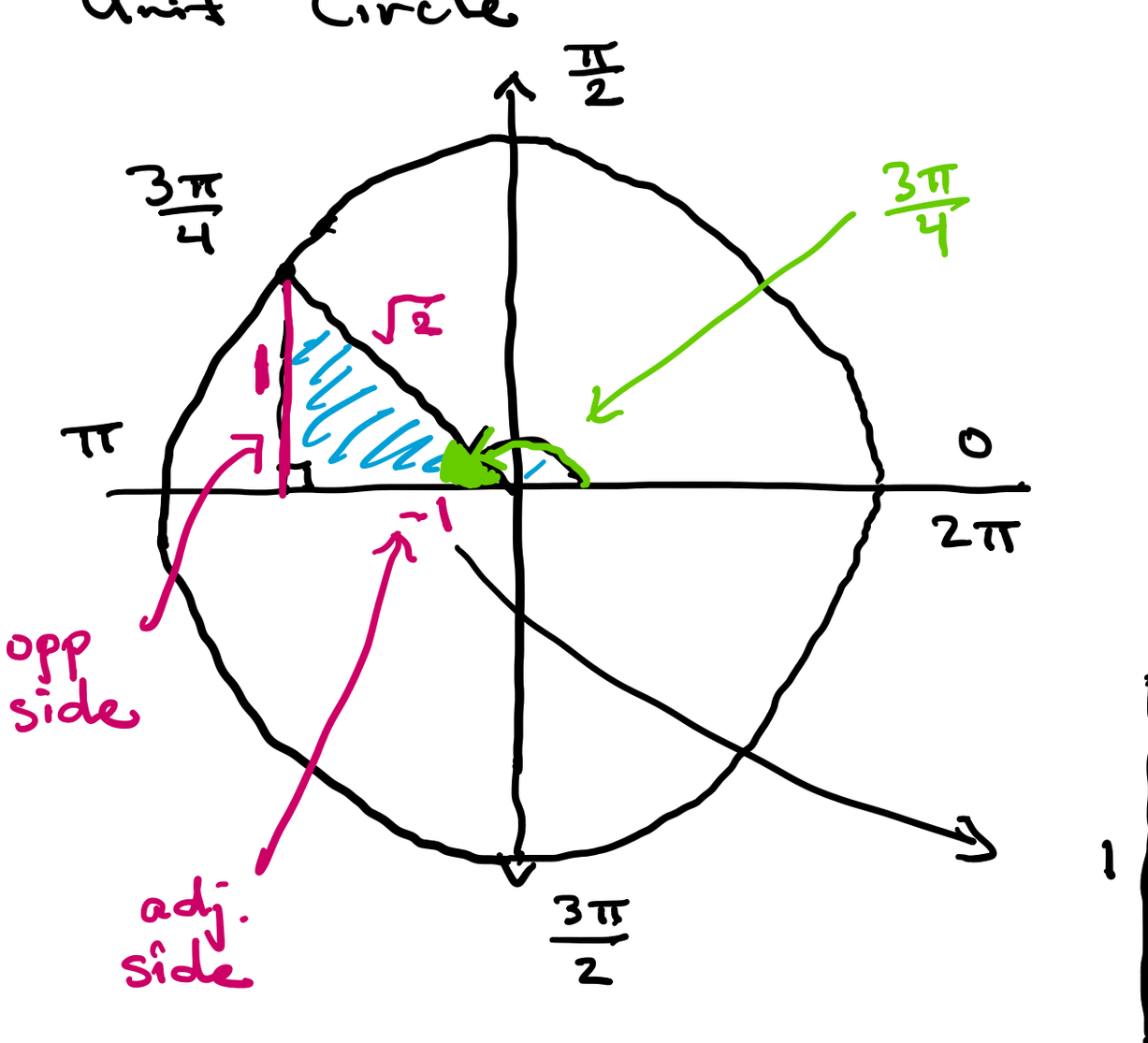
$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

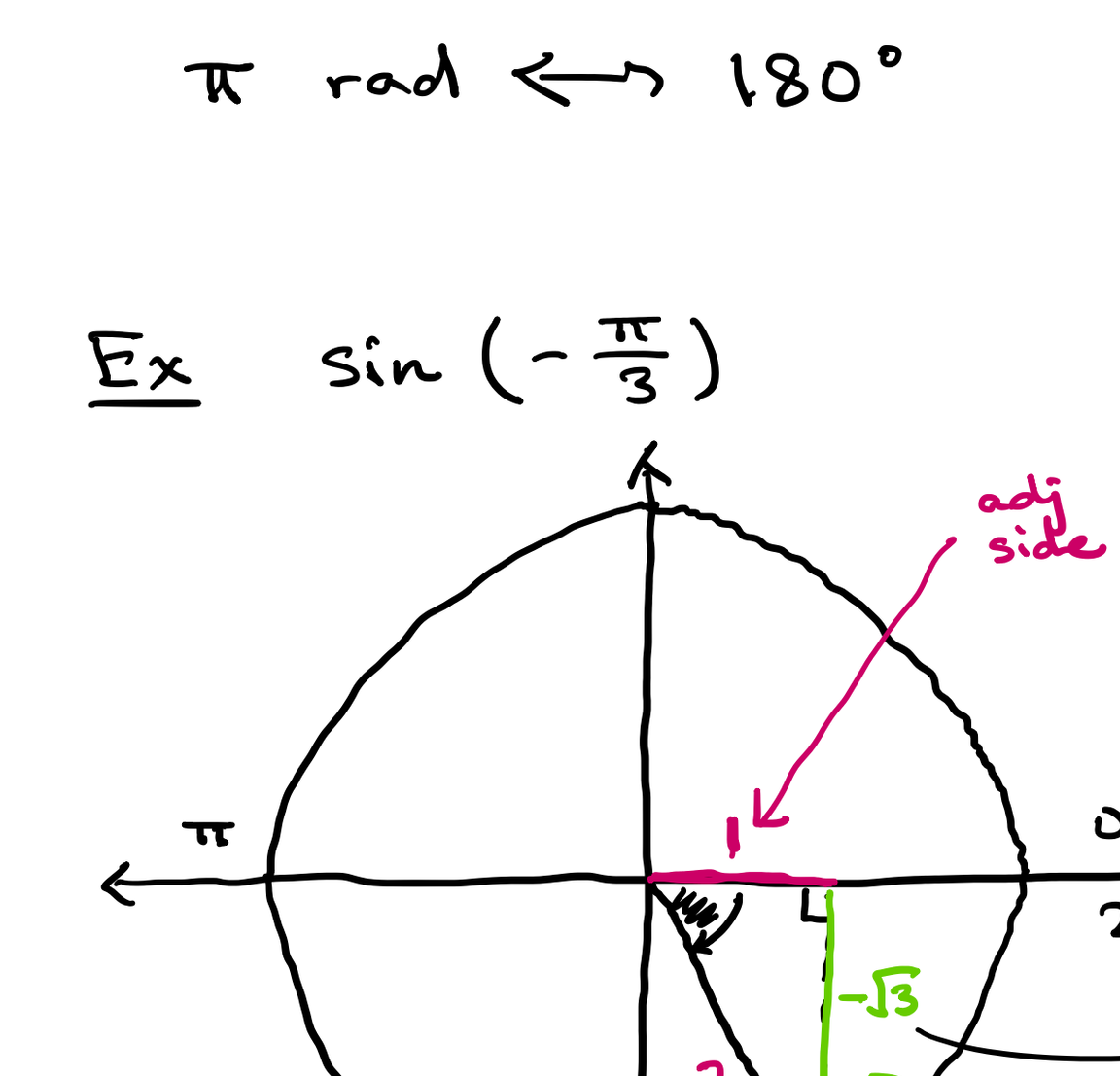
Graphs:





$\tan x = \frac{\sin x}{\cos x}$

Unit Circle

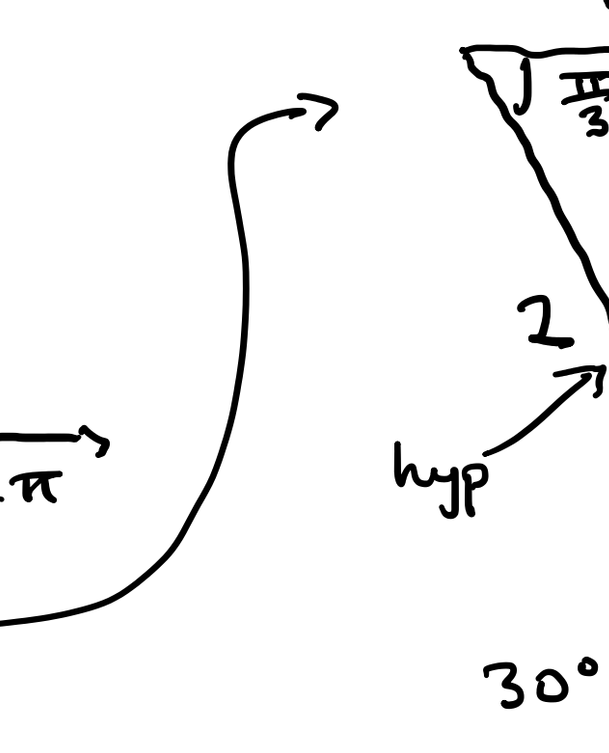


π rad ⟷ 180°

Ex $\sin\left(\frac{3\pi}{4}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

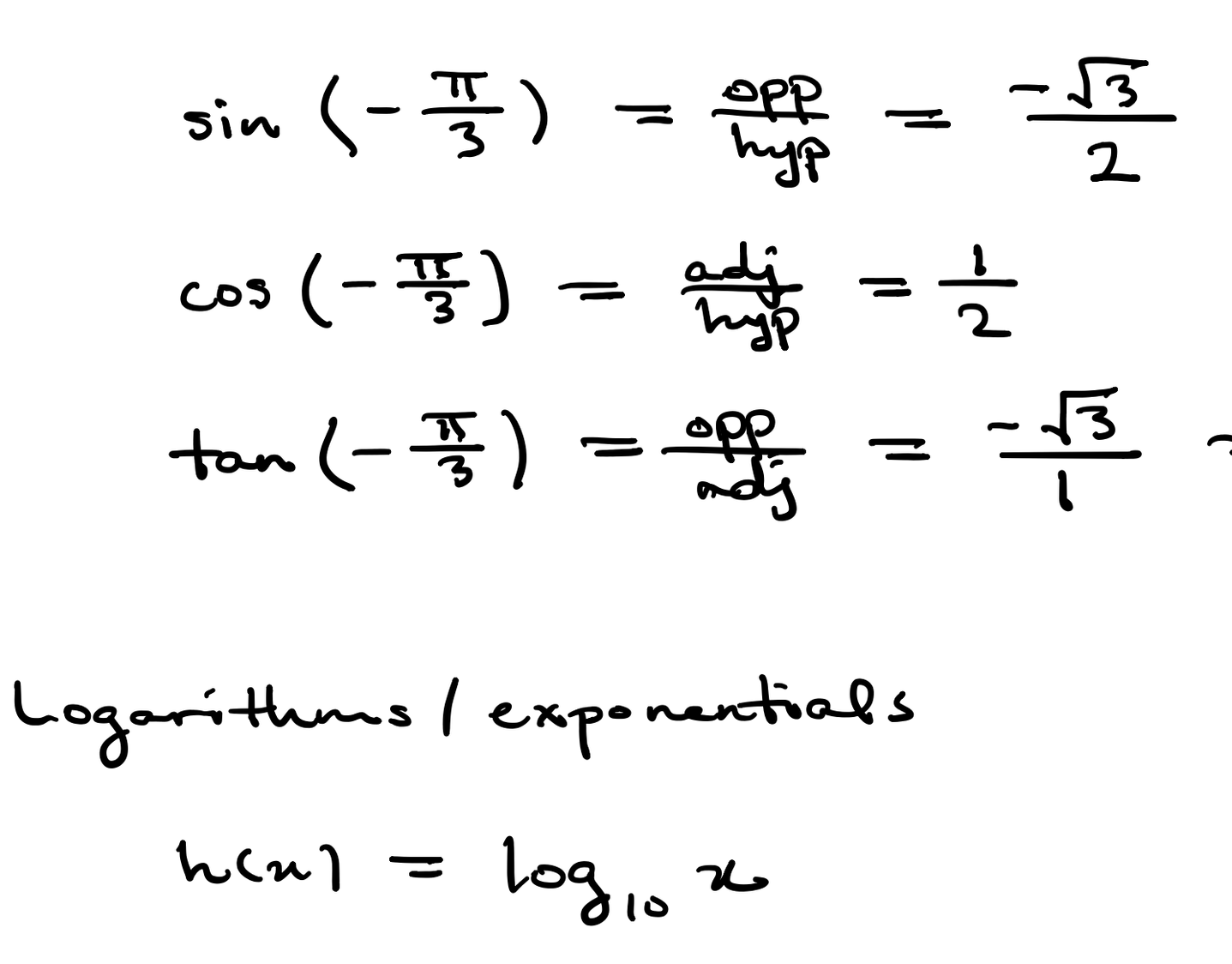
$\cos\left(\frac{3\pi}{4}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

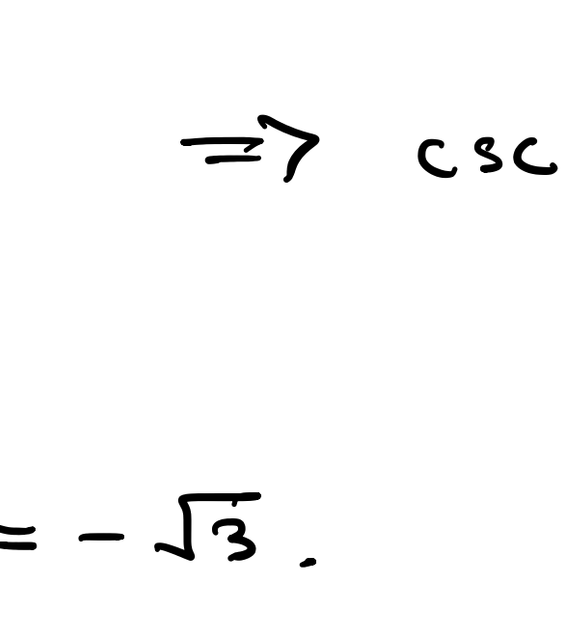
$\tan\left(\frac{3\pi}{4}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{-1} = -1$



$c^2 = 1^2 + 1^2 = 2$
$c = \sqrt{2}$

Ex $\sin\left(-\frac{\pi}{3}\right)$





$c^2 = a^2 + b^2$

$2^2 = 1^2 + b^2 \Rightarrow b^2 = 4 - 1 = 3$
$b = \sqrt{3}$

30° – 60° – 90°

$\sin\left(-\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{-\sqrt{3}}{2}$ $\Rightarrow \csc\left(-\frac{\pi}{3}\right) = \frac{1}{\sin\left(-\frac{\pi}{3}\right)} = -\frac{2}{\sqrt{3}}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$ ⋮

$\tan\left(-\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{adj}} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$.

Logarithms / exponentials

$h(x) = \log_{10} x$

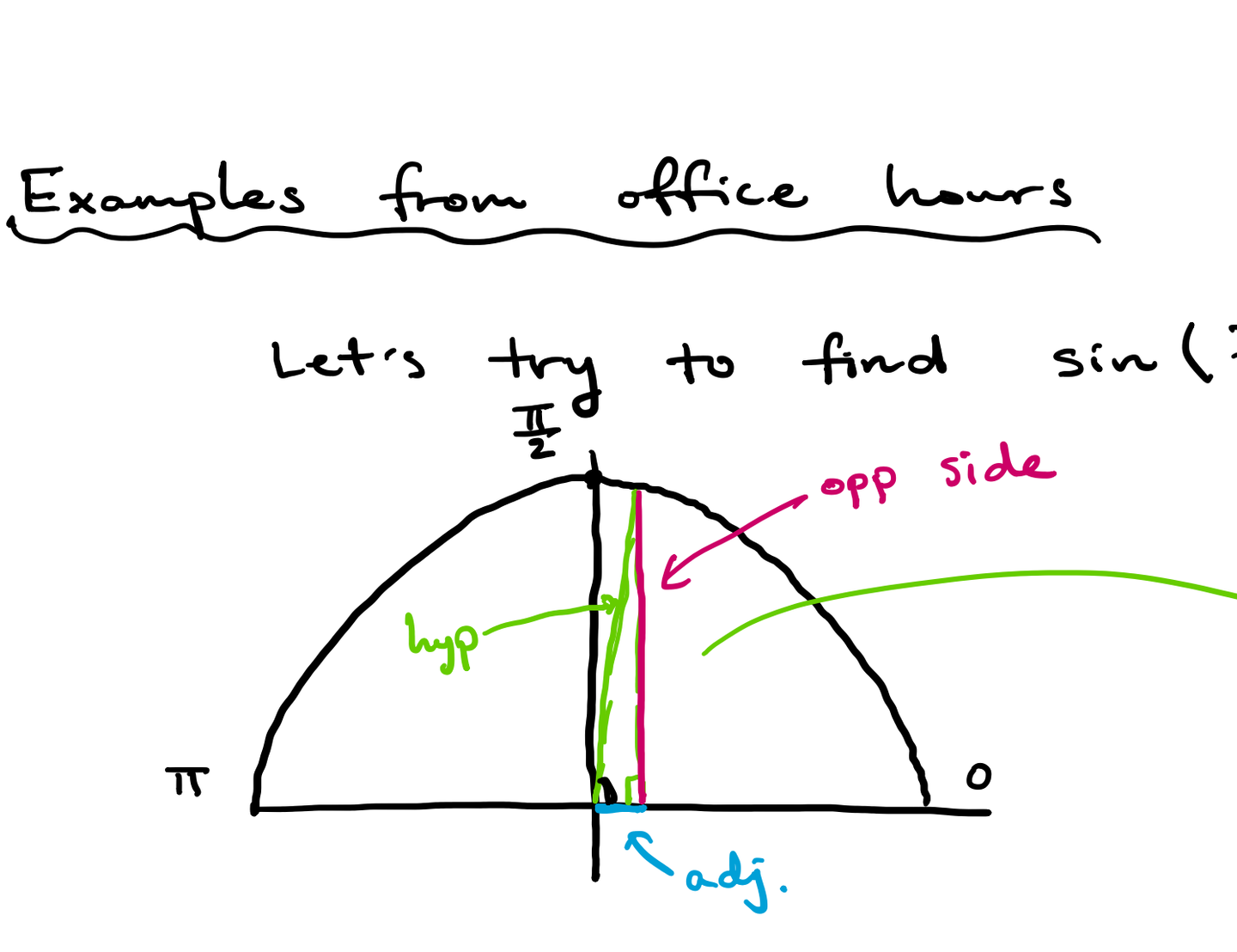
Notation: $\log_a b = c \iff a^c = b$.

(only makes sense if $a > 0$ and $b > 0$; ↑ a: base of logarithm)

e = natural log $\approx 2.71\ldots$

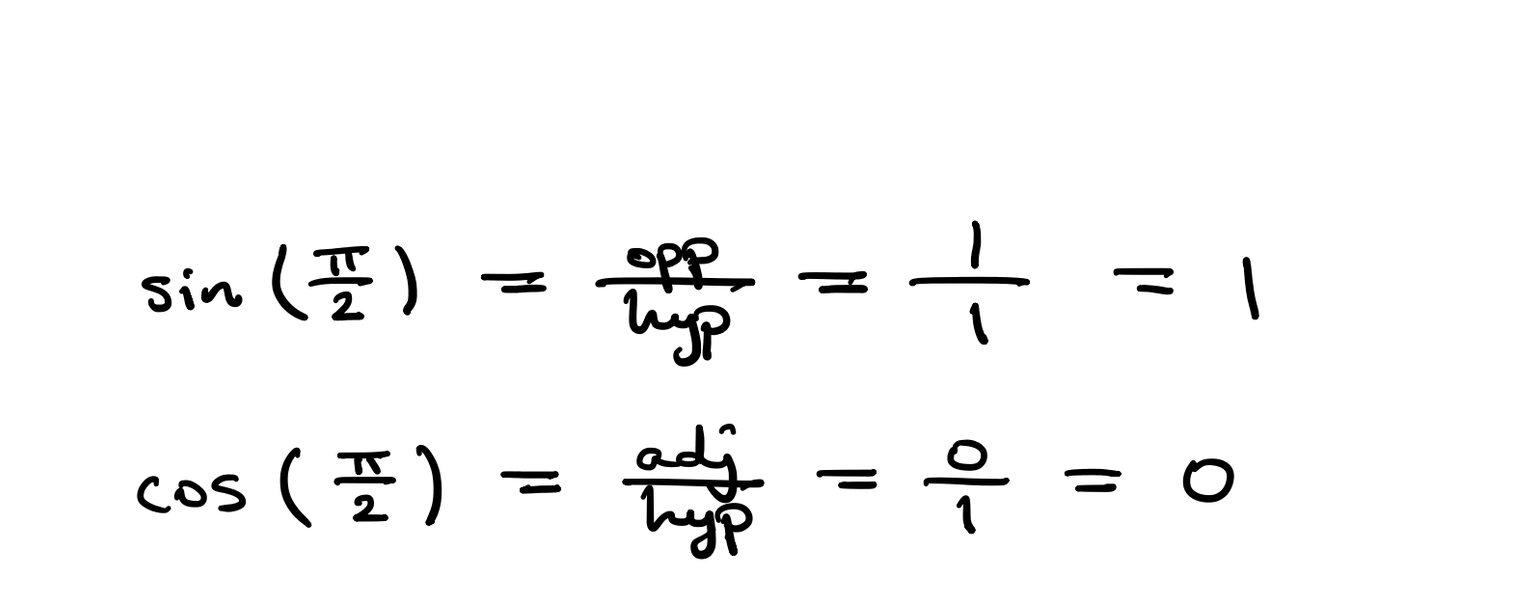
$h(x) = \log_e x = \ln x$ $g(x) = e^x$ exponential
↑ inverse of h

$\Rightarrow g(h(x)) = x = h(g(x))$



Examples from office hours

Let's try to find $\sin\left(\frac{\pi}{2}\right)$ and $\cos\left(\frac{\pi}{2}\right)$.



Think of triangle as vertical line

$\sin\left(\frac{\pi}{2}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{1} = 1$

$\cos\left(\frac{\pi}{2}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{0}{1} = 0$

$\tan\left(\frac{\pi}{2}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{0}$ = undefined or does not exist

Ex. Find $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\sin x} = \frac{0}{1} = 0$.

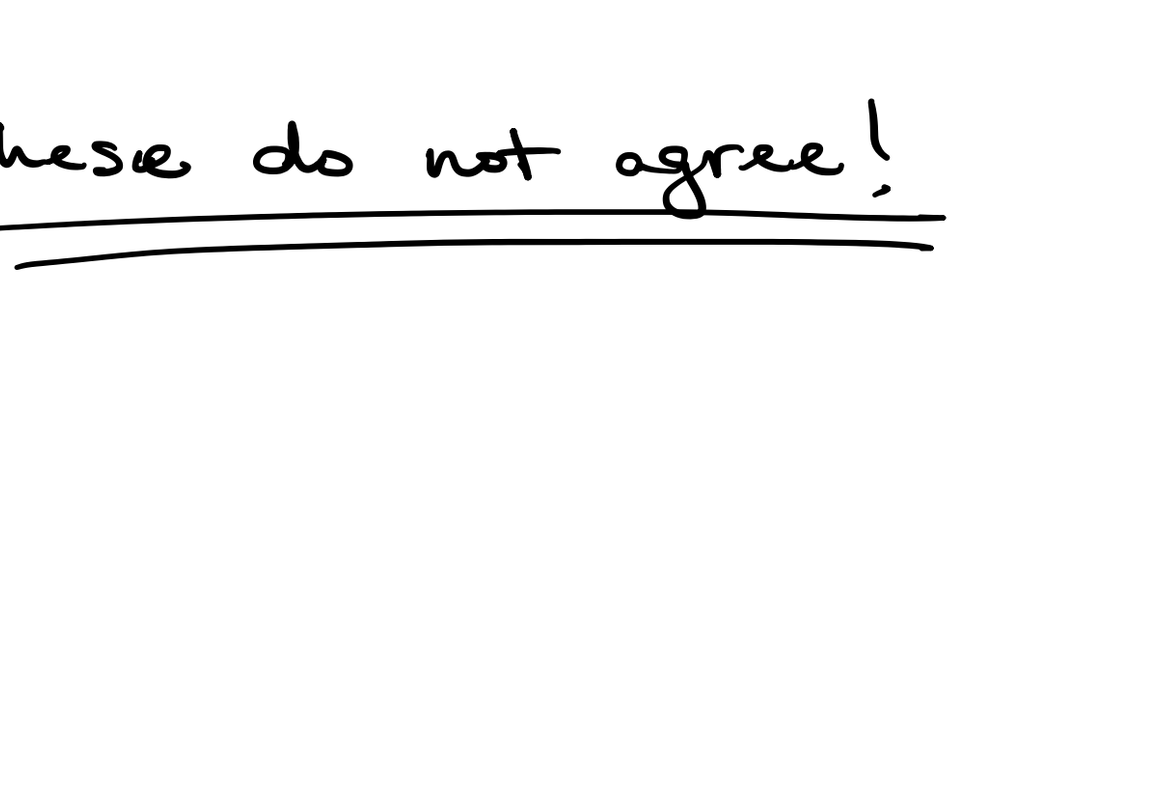
As $x \to \frac{\pi}{2}$, $\begin{cases} \cos x \to 0 \\ \sin x \to 1 \end{cases}$

Find $\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} \to \frac{1}{0}$ ← **Indeterminate**: need to find new method; could mean $+\infty$, $-\infty$, or DNE.

Try the following: break up into two limits

as $x \to \frac{\pi}{2}^+$, $\begin{cases} \cos x \to 0^- \\ \sin x \to 1 \end{cases}$

as $x \to \frac{\pi}{2}^-$, $\begin{cases} \cos x \to 0^+ \\ \sin x \to 1 \end{cases}$



So compute limits separately:

$\lim_{x \to \frac{\pi}{2}^+} \frac{\sin x}{\cos x} = \frac{1}{0^-} = -\infty$

$\lim_{x \to \frac{\pi}{2}^-} \frac{\sin x}{\cos x} = \frac{1}{0^+} = +\infty$

These do not agree!

This means that

$\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} = \text{DNE}$

Can also see this from $\frac{\sin x}{\cos x} = \tan x$ and graph of $\tan x$.

Ex Find $\lim_{x \to 3^+} \frac{1}{3 - x} = \frac{1}{0^-} = -\infty$ ($3 - x \to 0^-$)

Think of $3 - 3.1$
$3 - 3.01$
$3 - 3.001$
⋮

$\lim_{x \to 3^+} \frac{1}{x - 3} = +\infty$ ($x - 3 \to 0^+$)

Check: $\lim_{x \to 3} \frac{1}{3 - x} = \text{DNE}$