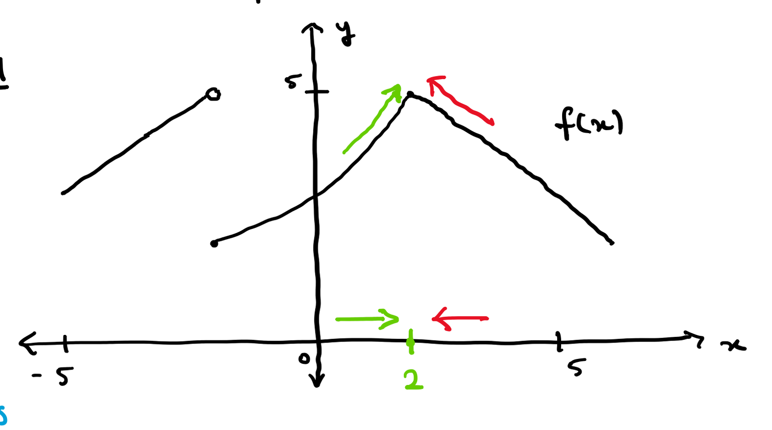
R09

Thursday, February 18, 2021   2:41 PM

OH: Today 6-7 pm.

Last week's quiz for R09

Q1



Pts

+2 (a) $\lim_{x \to 2^-} f(x) = 5$

+2 (b) $\lim_{x \to 2^+} f(x) = 5$ — answers agree

+2 (c) $\lim_{x \to 2} f(x) = 5$

Q2 A student computed $\lim_{x \to 4} \frac{3}{x-4} = \text{DNE}$.

Is this correct?

Solution: try to plug in $x=4$: $\frac{3}{4-4} = \frac{3}{0}$ ← doesn't make sense.

+2 { Compute one-sided limits.

$\lim_{x \to 4^-} \frac{3}{x-4} = \frac{3}{(\text{small negative } \#)} = -\infty$

↑ think of 3.9, 3.99, ...

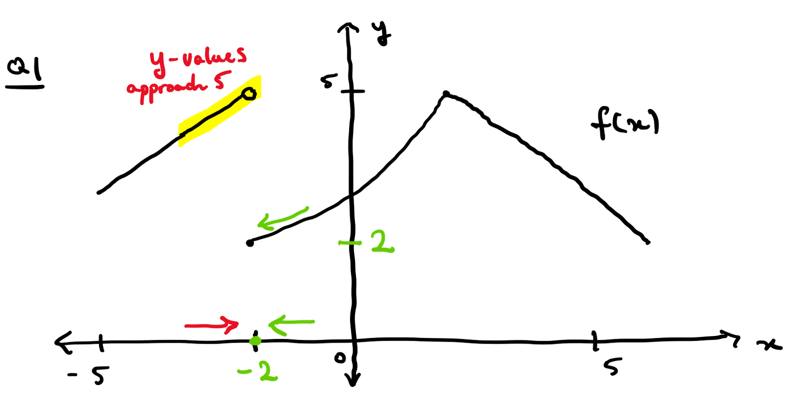
Similarly, $\lim_{x \to 4^+} \frac{3}{x-4} = \frac{3}{(\text{small } + \#)} = +\infty$ }

are different!

+2 So $\lim_{x \to 4} \frac{3}{x-4} = \text{DNE}$

Last week's quiz for R02

Q1



+2 (a) $\lim_{x \to -2^+} f(x) = 2$

+2 (b) $\lim_{x \to -2^-} f(x) = 5$ — different

+2 (c) $\lim_{x \to 2} f(x) = \text{DNE}$

Q2 A student computes $\lim_{x \to 2} \frac{3}{x^2-4} = \text{DNE}$. Is this correct?

Plugging in $x=2$ gives $\frac{3}{0}$ ← doesn't make sense

+2 { Consider $\lim_{x \to 2^+} \frac{3}{x^2-4} = \frac{3}{(\text{small } + \#)} = +\infty$

↑ think about $x = 2.1, 2.01, \ldots$

Similarly $\lim_{x \to 2^-} \frac{3}{x^2-4} = \frac{3}{(\text{small } - \#)} = -\infty$

↑ $x = 1.9, 1.99, \ldots$ }

are different!

+2 So $\lim_{x \to 2} \frac{3}{x^2-4} = \text{DNE}$.

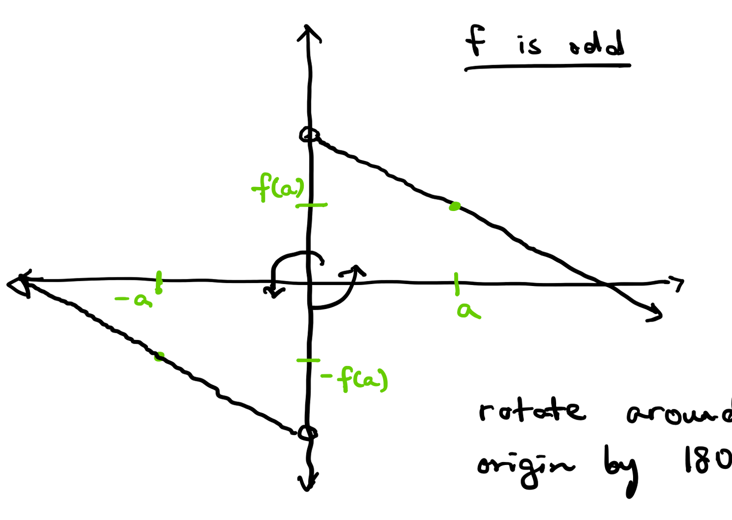
Review of even/odd functions

Def. (1) f is odd if $f(-x) = -f(x)$. **Ex** $f(x) = \sin x$, $f(x) = x, x^3$

(2) f is even if $f(-x) = f(x)$. **Ex.** $f(x) = \cos x$, $f(x) = x^2$

Graph

f is odd

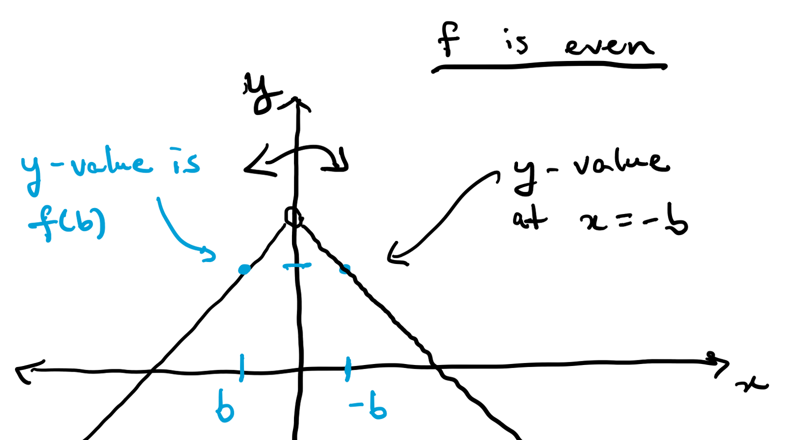


$f(-a) = -f(a)$

f is symmetric about the origin.

rotate around origin by 180°

f is even



$f(-b) = f(b)$

even functions are symmetric about the y-axis

Limits

(a) Evaluate limits by factoring and canceling

$\lim_{x \to -3} \frac{x^2+4x+3}{x^2-9}$ If you plug in $x=-3$: get $\frac{0}{0}$.

$= \lim_{x \to -3} \frac{(x+1)(x+3)}{(x+3)(x-3)}$

$= \lim_{x \to -3} \frac{x+1}{x-3} = \frac{-3+1}{-3-3} = \frac{-2}{-6} = \frac{1}{3}$.

↑ plug in $x=-3$

Ex 2:

$\lim_{x \to 3} \frac{2x^2-6x}{2x^2-5x-3} = \lim_{x \to 3} \frac{2x(x-3)}{(x-3)(2x+1)}$ → check $2x^2 \underbrace{-6x+x}_{-5x} -3$

$= \lim_{x \to 3} \frac{2x}{2x+1}$

plug in $= \frac{2(3)}{2(3)+1} = \frac{6}{7}$.

(b) Multiplying by conjugate "use this if your limit involves $\sqrt{\ }$"

$\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$ ← conjugate is $\sqrt{x-1}+2$

$= \lim_{x \to 5} \frac{(\sqrt{x-1}-2)\cdot(\sqrt{x-1}+2)}{(x-5)\cdot(\sqrt{x-1}+2)}$ ← factored version of a difference of squares

$= \lim_{x \to 5} \frac{(\sqrt{x-1})^2 - (2)^2}{(x-5)\cdot(\sqrt{x-1}+2)}$

$= \lim_{x \to 5} \frac{x-1-4}{(x-5)\cdot(\sqrt{x-1}+2)}$

$= \lim_{x \to 5} \frac{1}{\sqrt{x-1}+2} = \frac{1}{\sqrt{5-1}+2} = \frac{1}{2+2} = \frac{1}{4}$.

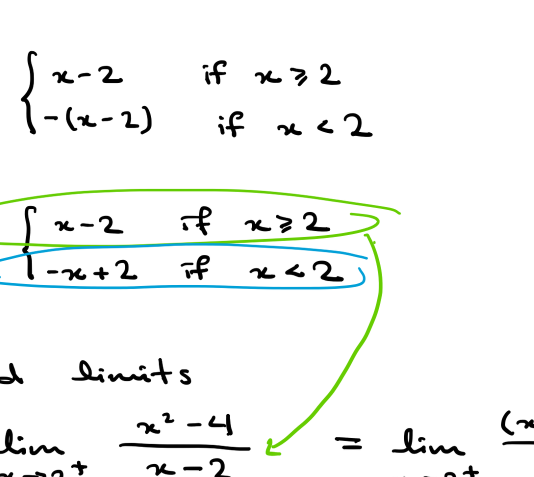
↑ plug in $x=5$

Office Hours:

Ex $\lim_{x \to 2} \frac{x^2-4}{|x-2|}$

↖ write as piecewise function.

$g(x) = |x-2|$



So

$g(x) = |x-2| = \begin{cases} x-2 & \text{if } x \geq 2 \\ -(x-2) & \text{if } x < 2 \end{cases}$

$= \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$

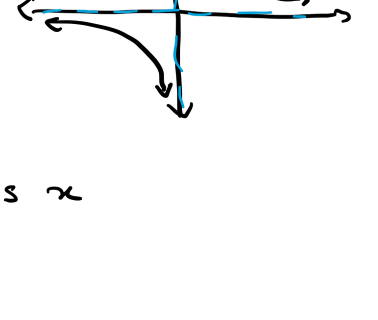
Consider one-sided limits

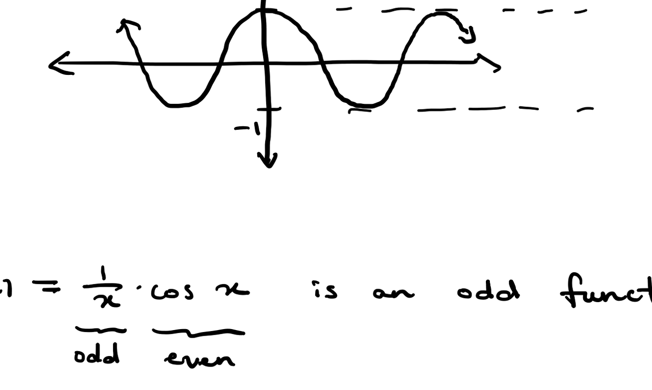
$\lim_{x \to 2^+} \frac{x^2-4}{|x-2|} = \lim_{x \to 2^+} \frac{x^2-4}{x-2} = \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2} = 4$

$\lim_{x \to 2^-} \frac{x^2-4}{|x-2|} = \lim_{x \to 2^-} \frac{x^2-4}{-x+2} = \lim_{x \to 2^-} \frac{(x+2)(x-2)}{-(x-2)} = \lim_{x \to 2^-} -(x+2) = -4$

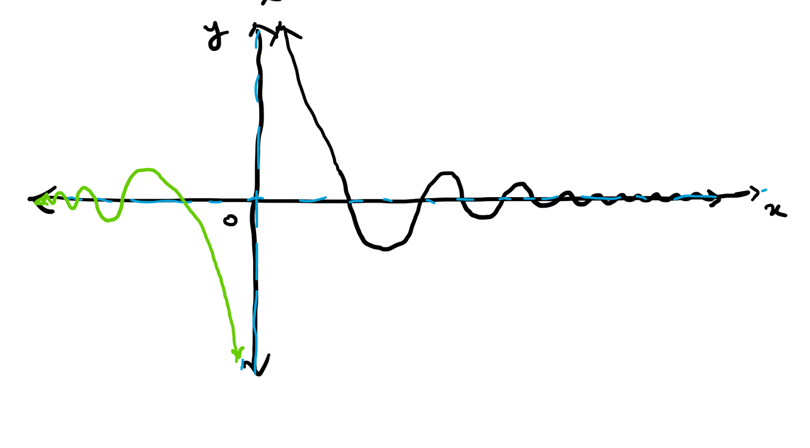
So $\lim_{x \to 2} \frac{x^2-4}{|x-2|} = \text{DNE}$,

Ex $f(x) = \frac{1}{x}$





$h(x) = \frac{1}{x} \cdot \cos x$



$h(x) = \underbrace{\frac{1}{x}}_{\text{odd}} \cdot \underbrace{\cos x}_{\text{even}}$ is an odd function.

(odd)·(even) = odd

(even)·(even) = even

(odd)·(odd) = even