my website: sites.google.com/stonybrook.edu/nathanchen/teaching

OH: today 6-7 pm

## Review of limits:

Intuition: suppose $x \to a$. What happens to $f(x)$.

If $f(x)$ is continuous, then

$$\lim_{x \to a} f(x) = f(a)$$

Examples where this is ok: $f$ is a polynomial, exponential, some trig, i.e. $\sin x$, $\cos x$.

This is not ok if $f(x) = \frac{1}{x-a}$

**Ex.** Consider $\lim_{x \to 0} \frac{1}{x}$. Plugging in gives $\frac{1}{0}$ indeterminate → find another way to take this limit.

Want to consider one-sided limits.

$\lim_{x \to 0^-} \frac{1}{x}$ and $\lim_{x \to 0^+} \frac{1}{x}$.

$x \to 0^-$: look at values of $x$ that are $< 0$

$x \to 0^+$: look at values of $x$ that are $> 0$.

To find $\lim_{x \to 0^-} \frac{1}{x}$, think about $x = -0.1, -0.01, -0.001, \ldots$

$$= \frac{1}{(\text{small negative } \#)}$$

$$= -\infty$$

$\frac{1}{x}$: $\frac{1}{-0.1} = -10$

$\frac{1}{-0.01} = -100$

$\frac{1}{-0.001} = -1000$

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{(\text{small positive } \#)}$$

$$= +\infty$$

$x = 0.1, 0.01, 0.001, \ldots$

$\frac{1}{x}$: $\frac{1}{0.1} = 10$

$\frac{1}{0.01} = 100$

$\frac{1}{0.001} = 1000$

are different!

Check whether $\lim_{x \to 0^-} \stackrel{?}{=} \lim_{x \to 0^+}$

If these are the same, then this becomes $\lim_{x \to 0}$.

If these are different, then $\lim_{x \to 0}$ DNE "does not exist".

So $\lim_{x \to 0} \frac{1}{x} = \text{DNE}$.

**Ex:** find $\lim_{x \to 0} \frac{1}{x^2}$.

Need to take one-sided limits.

$$\lim_{x \to 0^+} \frac{1}{x^2} = \frac{1}{(\text{small positive } \#)^2} = \frac{1}{(\text{small positive } \#)}$$

$$= +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x^2} = \frac{1}{(\text{small negative } \#)^2} = \frac{1}{(\text{small positive } \#)}$$

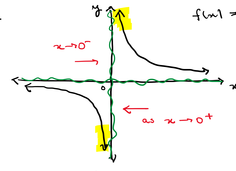
$$= +\infty.$$

agree.

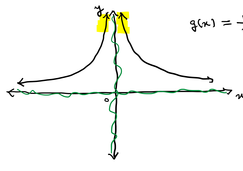
So $\lim_{x \to 0} \frac{1}{x^2} = +\infty$

**Graphs**

$f(x) = \frac{1}{x}$

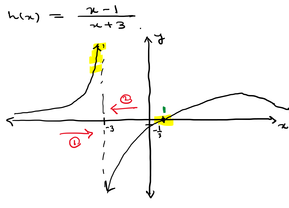



$g(x) = \frac{1}{x^2}$




$\lim_{x \to 0} \frac{1}{x^2} = +\infty$

**Ex.** $h(x) = \frac{x-1}{x+3}$.



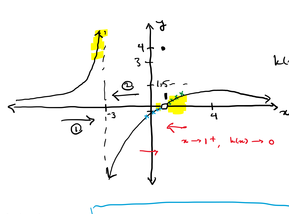

① What is $\lim_{x \to -3^-} h(x)$? $+\infty$

② What is $\lim_{x \to -3^+} h(x)$? $-\infty$

③ What is $\lim_{x \to -3} h(x)$? DNE because $\lim_{x \to -3^-} \neq \lim_{x \to -3^+}$.

④ What is $\lim_{x \to 1} h(x)$?

$$= \frac{1-1}{1+3} = \frac{0}{4} = 0.$$




$k(x)$. $k(1) = 4$

What is $\lim_{x \to 1} k(x)$? still 0.

$\lim_{x \to 1^+} k(x) = 0$ and $\lim_{x \to 1^-} k(x) = 0$. are the same!