3/31/2020 R03

Tuesday, March 31, 2020 5:23 PM

Announcements - Office hours through Zoom
Th 4:30 - 5:30 pm (I'll send out email)

- website for recitation
sites.google.com/stonybrook.edu/nathanchen/teaching

Integration by parts

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

Helpful: $\int (\text{polynomial}) \cdot (\text{exponential}/\text{trig}/\log)$

Ex. $\int x \cdot e^{3x} dx$

Choose $u = x$, $dv = e^{3x} \cdot dx$

$du = dx$, $v = \int e^{3x} dx = \frac{1}{3} e^{3x}$

$$= u \cdot v - \int v \cdot du$$
$$= x \cdot \frac{1}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx$$
$$= \frac{1}{3} \cdot x \cdot e^{3x} - \frac{1}{3} \cdot \int e^{3x} dx$$
$$= \frac{1}{3} \cdot x \cdot e^{3x} - \underbrace{\frac{1}{3} \cdot \frac{1}{3}}_{\frac{1}{9}} e^{3x} + C$$

Ex. $\int \ln x \cdot dx$

Choose $u = \ln x$, $dv = dx$

$du = \frac{1}{x} \cdot dx$, $v = \int dx = x$.

$$= u \cdot v - \int v \cdot du$$
$$= x \cdot \ln x - \int \cancel{x} \cdot \frac{\overset{1}{\cancel{1}}}{\cancel{x}} dx$$
$$= x \cdot \ln x - x + C.$$

Ex $\int_{1/e}^{1} \ln x \cdot dx = (x \cdot \ln x - x) \Big|_{1/e}^{1}$

$$= (1 \cdot \overset{0}{\cancel{\ln 1}} - 1) - \left(\frac{1}{e} \cdot \ln\left(\frac{1}{e}\right) - \frac{1}{e}\right)$$

cancelled

$$= -1 - \left(\frac{1}{e} \cdot (-1) - \frac{1}{e}\right)$$
$$= -1 + \frac{2}{e}.$$

$\ln\left(\frac{1}{e}\right)$
$= \ln(e^{-1})$
$= (-1) \cdot \ln(e)$
$= -1$

Ex $\int \frac{\ln^2(x)}{x} \cdot dx$ ($\frac{1}{x} \cdot dx = du$)

We can use u-sub!

Choose $u = \ln x$, $du = \frac{1}{x} \cdot dx$

$$= \int u^2 \cdot du$$

power rule

$$= \frac{u^3}{3} + C$$
$$= \frac{(\ln x)^3}{3} + C.$$

Ex May need to use integration by parts several times

$\int x^3 \cdot \cos x \cdot dx$ (deg 3)

$u = x^3$, $dv = \cos x \cdot dx$

$du = 3x^2 dx$, $v = \int \cos x \, dx = \sin x$

IBP

$$= u \cdot v - \int v \cdot du$$
$$= x^3 \cdot \sin x - \int \sin x \cdot 3x^2 \, dx$$

(deg 2)

IBP

$\int (\text{deg } 1) \cdot \cos x \cdot dx$

IBP

$\int (\text{constant}) \sin x \cdot dx$

Ex $\int x \cdot \sec^2 x \cdot dx$

Choose $u = x$, $dv = \sec^2 x \cdot dx$

$du = dx$, $v = \int \sec^2 x \cdot dx = \tan x$

$$= u \cdot v - \int v \cdot du$$
$$= x \cdot \tan x - \int \tan x \cdot dx$$

$\int \tan x \cdot dx = \int \frac{\sin x}{\cos x} \cdot dx = -\ln|\cos x|$

let $s = \cos x$, $ds = -\sin x \cdot dx$

Use u-subsitution

$$= x \cdot \tan x + \ln|\cos x| + C$$

Trig substitution

Ex $\int \sin^4 x \cdot \cos x \cdot dx$

Try u-sub

$u = \sin x$, $du = \cos x \cdot dx$

$$= \int u^4 \cdot du$$
$$= \frac{u^5}{5} + C$$
$$= \frac{\sin^5 x}{5} + C.$$

Ex $\int \sin^4 x \cdot \cos^5 x \cdot dx$ (odd)

$\sin^2 x + \cos^2 x = 1$

Apply $\cos^2 x = 1 - \sin^2 x$

$$= \int \sin^4 x \cdot (\cos^2 x)^2 \cdot \cos x \cdot dx$$
$$= \int \sin^4 x \cdot (1 - \sin^2 x)^2 \underbrace{\cos x \cdot dx}_{= du}$$

Use u-sub
$u = \sin x$
$du = \cos x \cdot dx$

$$= \int u^4 \cdot (1 - u^2)^2 \cdot du$$
$$= \int u^4 \cdot (1 - 2u^2 + u^4) \cdot du$$
$$= \int (u^4 - 2u^6 + u^8) \, du$$

use power rule term by term

$$= \frac{u^5}{5} - 2 \cdot \frac{u^7}{7} + \frac{u^9}{9} + C$$

subsitute $u = \sin x$, $+ C$

Given $\int \sin^k x \cdot \cos^l x \cdot dx$

a) If l = odd #, use $\cos^2 x = 1 - \sin^2 x$

keep a factor of $\cos x \cdot dx$

use u-sub with $u = \sin x$

b) If k = odd #, use $\sin^2 x = 1 - \cos^2 x$

keep a factor of $\sin x \cdot dx$

use u-sub with $u = \cos x$

c) If k, l both even ... use double-angle formulas.