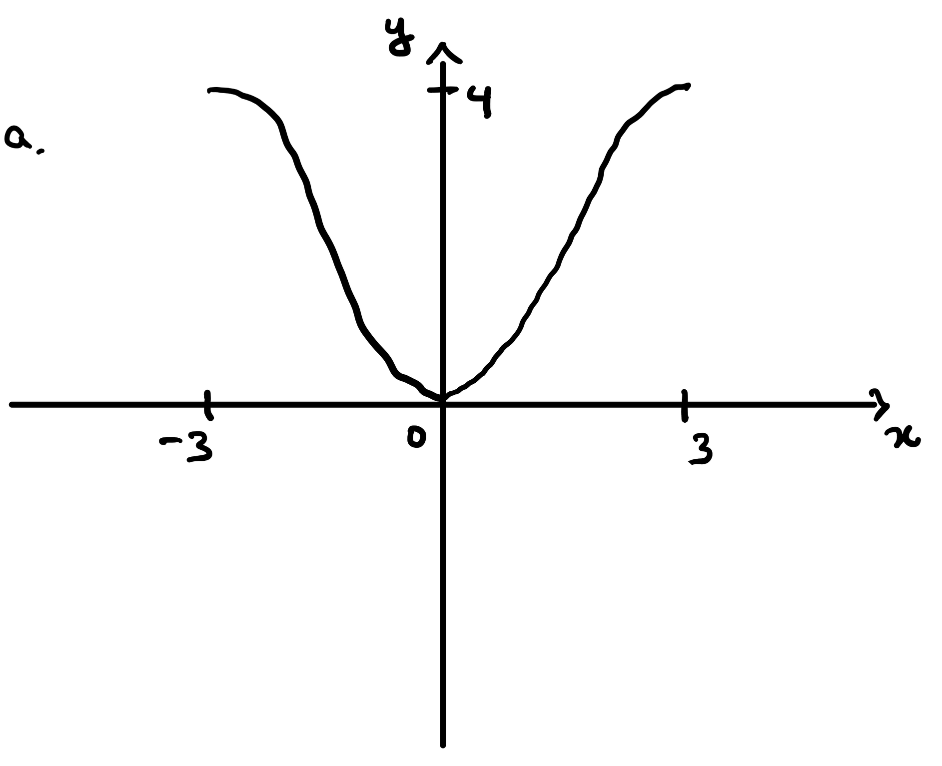
R02

Thursday, February 25, 2021 4:46 PM

a.



f is even

$\downarrow$

f is symmetric when reflecting across y-axis

b. A student computed $\lim_{x\to 3} \frac{x^2-x-6}{2(x-3)} = \text{DNE}$.

Plugging $x=3$, we get $\frac{0}{0}$

$$\lim_{x\to 3} \frac{x^2-x-6}{2(x-3)} = \lim_{x\to 3} \frac{(x+2)\cancel{(x-3)}}{2\cancel{(x-3)}}$$

$$= \lim_{x\to 3} \frac{x+2}{2} = \frac{3+2}{2} = \frac{5}{2}$$

plug in $x=3$

Warm-up: $\lim_{x\to -3} \frac{\sqrt{x+4}-1}{x+3}$ Hint: use conjugate

conjugate = $\sqrt{x+4}+1$ or $-\sqrt{x+4}-1$

$$= \lim_{x\to -3} \frac{\sqrt{x+4}-1}{x+3} \cdot \boxed{\frac{\sqrt{x+4}+1}{\sqrt{x+4}+1}}$$

$$= \lim_{x\to -3} \frac{(\sqrt{x+4})^2 - (1)^2}{(x+3)\cdot(\sqrt{x+4}+1)}$$

$$= \lim_{x\to -3} \frac{\cancel{x+4-1}}{\cancel{(x+3)}\cdot(\sqrt{x+4}+1)}$$

$$= \lim_{x\to -3} \frac{1}{\sqrt{x+4}+1} = \frac{1}{\sqrt{-3+4}+1} = \frac{1}{1+1} = \frac{1}{2}.$$

Ex $\lim_{x\to -3} \frac{\frac{1}{x+2}+1}{x+3} = \lim_{x\to -3} \frac{\frac{1}{x+2}+\frac{x+2}{x+2}}{x+3}$

$$= \lim_{x\to -3} \frac{\left(\frac{x+3}{x+2}\right)}{x+3}$$

$$= \lim_{x\to -3} \left(\frac{\cancel{x+3}}{x+2}\right)\cdot\frac{1}{\cancel{x+3}}$$

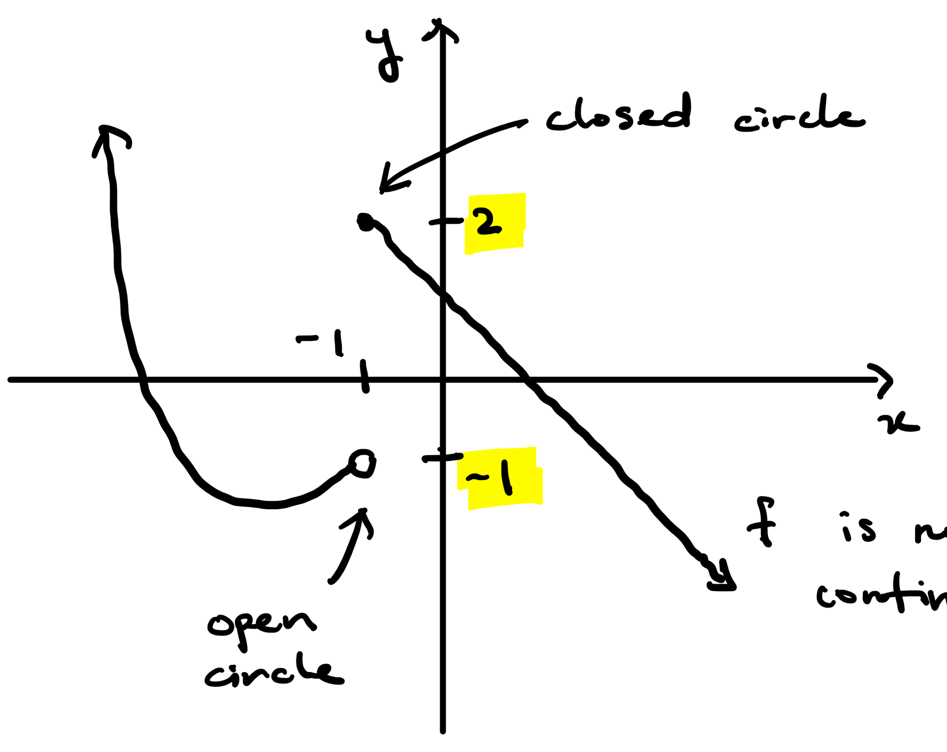
$$= \lim_{x\to -3} \frac{1}{x+2} = \frac{1}{-3+2} = -1$$

Def. A function f is continuous at $x=a$ if

$$\underbrace{\lim_{x\to a} f(x)}_{} = f(a)$$

only defined if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$



$\lim_{x\to -1} f(x) = \text{DNE}$,

$\neq f(-1) = 2$

Ex $g(u) = \begin{cases} \boxed{\frac{6u^2+u-2}{2u-1}} & \text{if } u \neq \frac{1}{2} \\ 7/2 & \text{if } u = \frac{1}{2} \end{cases}$

Is g continuous at $u = \frac{1}{2}$?

Need to check $\lim_{u\to\frac{1}{2}} g(u) = g(\frac{1}{2})$?

$$\boxed{\lim_{u\to\frac{1}{2}} \frac{6u^2+u-2}{2u-1} = \frac{7}{2}\ ?}$$

this is true!

$$= \lim_{u\to\frac{1}{2}} \frac{\cancel{(2u-1)}(3u+2)}{\cancel{2u-1}}$$

So g is continuous at $u=\frac{1}{2}$.

$$= \lim_{u\to\frac{1}{2}} 3u+2$$

$$= \frac{3}{2}+2 = \frac{7}{2} \checkmark$$

Ex $f(x) = \begin{cases} \sqrt{kx} & \text{if } 0 \le x \le 3, \\ x+1 & \text{if } 3 < x \le 10, \end{cases}$

For which values of k is f continuous on $[0,10]$?

Want f to be continuous at $x=3$. $f(3) = \sqrt{k\cdot 3}$

Want $\lim_{x\to 3} f(x) \overset{?}{=} f(3) = \sqrt{k\cdot 3}$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \sqrt{k\cdot x} = \sqrt{k\cdot 3}$ $\checkmark$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+1 = 4$

both agree

Set $\sqrt{k\cdot 3} = 4 \Rightarrow 3k = 16 \Rightarrow \boxed{k = \frac{16}{3}}$