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Final Exam Review

Here are some review problems for the final. Please send me an email if you find any errors.

(∗) Denotes a challenging problem.

1. (Warm-up) Arrange the following 5 numbers in increasing order (from smallest to largest):

$$\sec(\pi/4), \quad 3 \cdot \arctan\left(\sqrt{3}\right), \quad -\tan(-\pi/3),$$

$$\int_0^{1/4} x^{-1/2} dx, \quad \lim_{x \to -\infty} 2^{k(x)},$$

given $k(x) = \frac{x^2-2x+1}{x^2-4}$.

2. Solve the following integrals:

(a)

$$\int \frac{\sqrt{x}}{x^{3/2}-1} dx.$$

(b)

$$\int_{1/3}^{1} \cos(3\pi(x-1)) dx.$$

(c)

$$\int \left(\frac{3-x}{x}\right)^2 dx.$$

(d) (∗)

$$\int_{-3}^{0} (x+5)\sqrt{9-x^2} dx.$$

3. Use integration by parts:

(a)

$$\int x^2 \cos 3x dx.$$

(b)

$$\int \arctan x dx.$$

(c) (∗)

$$\int \sin(\ln(2x))dx.$$

4. Trig integrals:

(a)

$$\int \tan^3 x \sec^3 x dx.$$

(b)

$$\int \sin^2 x \cos^2 x dx.$$

(c) (∗)

$$\int \frac{\tan^3 x}{\sqrt{\sec x}} dx.$$

5. Trig substitution:

(a)

$$\int \sqrt{16 - x^2} dx.$$

(b)

$$\int \frac{\sqrt{x^2 - 25}}{x} dx.$$

(c)

$$\int \frac{dx}{(1 + x^2)^{3/2}}.$$

6. Partial fraction decomposition:

(a)

$$\int \frac{2x^4}{x^2 - 2x} dx.$$

(b)

$$\int \frac{1}{x^4 - 1} dx.$$

7. Improper integrals:

(a)

$$\int_2^4 \frac{dx}{(x - 3)^2}.$$

(b)
$$\int_0^\infty e^{-x}\cos x dx.$$

(c)
$$\int_{-\infty}^\infty \frac{e^x}{1+e^{2x}}dx.$$

8. Area between curves (try to draw a picture first):
   (a) Find the area bounded between $f(x) = x^3$ and $g(x) = x^2 + 2x$.
   (b) Find the area bounded between $f(x) = 2x^2$, $g(x) = 4 - 2x$, and the x-axis by integrating with respect to y.
9. Volume of solids (see pg. 163 of the calculus book, volume 2 to compare the different methods):
   (a) Let R be the region bounded by the graph of $y = \sqrt{4 - x^2}$, $y = 0$, and $x = 0$. Use the disk method to find the volume of the solid of revolution generated by rotating R around the y-axis.
   (b) Let R be the region bounded by the graph of $y = \sin x$, $y = 5\sin x$, $x = 0$, and $x = \pi$. Use the washer method to find the volume when the region is revolved around the x-axis.
   (c) Consider the region bounded between $y = \sqrt[5]{x}$, the x-axis, $x = 0$, and $x = 1$. Use the shell method to find the volume of the solid obtained by rotating the region around the y-axis.
10. Find the length of the function $y = -\frac{1}{2}x + 2$ from $x = 1$ to $x = 4$, and verify this using geometry.