Office hours 4:30pm on 4/3/2020

Friday, April 3, 2020 4:34 PM

Ex $\int_1^e 9\cdot t^2\cdot \ln t\cdot dt$

IBP: $\int u\cdot dv = u\cdot v - \int v\cdot du$

Choose $\ln t = u$, $dv = 9t^2\cdot dt$.

$du = \frac{1}{t}\,dt$

$v = \int 9t^2\cdot dt = 3t^3$

$= \left(uv - \int v\cdot du\right)\Big|_1^e$

$= \left(3t^3\cdot \ln t - \int 3t^{3}\cdot \frac{1}{t}\,dt\right)\Big|_1^e$

$= \left(3t^3\cdot \ln t - \int 3t^2\cdot dt\right)\Big|_1^e$

$= \left(3t^3\cdot \ln t - t^3\right)\Big|_1^e$

$= (3e^3\cdot \ln e - e^3) - (3\cdot 1^3\cdot \ln 1 - 1^3)$ (with $\ln e = 1$, $\ln 1 = 0$)

$= (2e^3) - (-1)$

$= 2e^3 + 1$.

$\log_e e = \ln e = z \iff e^z = e = e^1 \iff z = 1$

$\log_e 1 = z \iff e^z = 1 = e^0 \iff z = 0$.

Ex $\int \cos(5x)\,dx$

Try u-sub: $u = 5x$, $du = 5\cdot dx$, $dx = \frac{1}{5}\,du$

$= \int \cos u\cdot \frac{1}{5}\,du$

$= \frac{1}{5}\cdot \int \cos u\cdot du$

$= \frac{1}{5}\cdot \sin u + C$

$= \frac{1}{5}\cdot \sin(5x) + C$.

Ex $\int \cos^7 x\,dx = \int \cos^6 x\cdot \cos x\cdot dx = \int (\cos^2 x)^3\cdot \cos x\cdot dx$

Ex $\int \cos^3 x\cdot dx$

$= \int \cos^2 x\cdot \cos x\cdot dx$

$= \int (1-\sin^2 x)\cdot \cos x\cdot dx$

$= \int (1-u^2)\,du$

$= u - \frac{u^3}{3} + C$

$= \sin x - \frac{\sin^3 x}{3} + C$

* $\cos^2 x = 1 - \sin^2 x$.

u-sub: $u = \sin x$, $du = \cos x\cdot dx$

Ex. $\int \cos^4 x\cdot dx$

Double-angle identity:

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$ (using $\sin^2 x = 1 - \cos^2 x$)

$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$= \int (\cos^2 x)^2\,dx$

$= \int \left(\frac{1}{2}(\cos 2x + 1)\right)^2 dx$

$= \frac{1}{4}\cdot \int (\cos 2x + 1)^2\cdot dx$

$= \frac{1}{4}\cdot \int [\cos^2(2x) + 2\cdot \cos(2x) + 1]\,dx$ — can integrate

$\cos^2(2x) = \frac{1}{2}(\cos 4x + 1)$

Ex $\int \frac{4\,dx}{x^2\cdot\sqrt{x^2+49}}$ with $x = 7\cdot \tan\theta \Rightarrow dx = 7\cdot \sec^2\theta\cdot d\theta$

$= 4\cdot \int \frac{7\cdot \sec^2\theta\cdot d\theta}{49\cdot \tan^2\theta\cdot \sqrt{49\cdot \tan^2\theta + 49}}$

$\sqrt{49\cdot ?} = \sqrt{49}\cdot\sqrt{?} = 7\cdot\sqrt{?}$

$= \frac{4}{7}\cdot \int \frac{\sec^2\theta\cdot d\theta}{\tan^2\theta\cdot 7\cdot\sqrt{\tan^2\theta + 1}}$

$\frac{1}{\cos^2\theta}(\sin^2\theta + \cos^2\theta) = 1\cdot \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$= \frac{4}{49}\cdot \int \frac{\sec^2\theta\cdot d\theta}{\tan^2\theta\cdot \sec\theta}$

$= \frac{4}{49}\cdot \int \frac{\sec\theta}{\tan^2\theta}\cdot d\theta$

$\sec\theta = \frac{1}{\cos\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{4}{49}\cdot \int \frac{1}{\cos\theta}\cdot \frac{\cos^2\theta}{\sin^2\theta}\cdot d\theta$

$= \frac{4}{49}\cdot \int \frac{\cos\theta}{\sin^2\theta}\cdot d\theta$

Try u-sub: $u = \sin\theta$, $du = \cos\theta\cdot d\theta$

$= \frac{4}{49}\cdot \int \frac{1}{u^2}\cdot du$ ($= u^{-2}$)

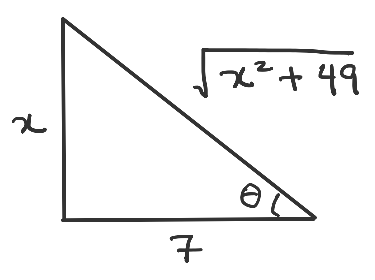
$= \frac{4}{49}\cdot (-1)\cdot u^{-1} + C$

$= -\frac{4}{49}\cdot \frac{1}{\sin\theta} + C$

$= -\frac{4}{49}\cdot \frac{\sqrt{x^2+49}}{x} + C$

Recall $x = 7\cdot \tan\theta$

$\tan\theta = \frac{x}{7}$



Double-angle formulas:

$\cos(2x) = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$ ($\sin^2 x = 1 - \cos^2 x$)

$= 1 - 2\cdot \sin^2 x$ ($\cos^2 x = 1 - \sin^2 x$)

Two formulas:

$\cos^2 x = \frac{1}{2}\cdot(\cos(2x) + 1)$

$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

Useful for $\int \cos^4 x\,dx$, $\int \sin^6 x\,dx$, even powers...

Ex $\int \frac{x^3}{\sqrt{x^2+81}}\cdot dx$ $\quad x = 9\tan\theta \quad dx = 9\sec^2\theta\cdot d\theta$

$= \int \frac{9^3\tan^3\theta}{\sqrt{81\cdot \tan^2\theta + 81}}\cdot 9\sec^2\theta\cdot d\theta$

$= 9^4\cdot \int \frac{\tan^3\theta\cdot \sec^2\theta}{9\cdot\sqrt{\tan^2\theta + 1}}\,d\theta$ (with $\tan^2\theta + 1 = \sec^2\theta$)

$= 9^3\cdot \int \tan^3\theta\cdot \sec\theta\cdot d\theta$

Try u-sub: $u = \sec\theta$

$du = \tan\theta\cdot \sec\theta\cdot d\theta$

use $\tan^2\theta = \sec^2\theta - 1$

$= 9^3\cdot \int \tan^2\theta\cdot \tan\theta\cdot \sec\theta\cdot d\theta$ ($\tan\theta\cdot\sec\theta\cdot d\theta = du$)

$= 9^3\cdot \int (\sec^2\theta - 1)\cdot \tan\theta\cdot \sec\theta\cdot d\theta$

$= 9^3\cdot \int (u^2 - 1)\cdot du$

$= 9^3\cdot \left(\frac{u^3}{3} - u\right) + C$

$= 9^3\cdot \left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C$

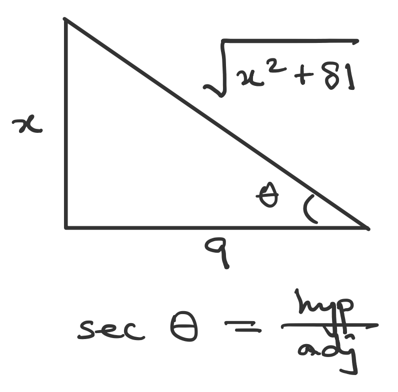
$= 9^3\cdot \left(\frac{\frac{(x^2+81)^{3/2}}{9^3}}{3} - \frac{\sqrt{x^2+81}}{9}\right) + C$

$= \frac{(x^2+81)^{3/2}}{3} - 9^2\cdot\sqrt{x^2+81} + C$

Recall:

$x = 9\cdot \tan\theta$

$\tan\theta = \frac{x}{9} = \frac{\text{opp}}{\text{adj.}}$



$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+81}}{9}$

Ex. $\int \sec^9 x\cdot \tan^3 x\cdot dx$

$= \int \sec^8 x\cdot \tan^2 x\cdot \sec x\cdot \tan x\cdot dx$ ($\tan^2 x = \sec^2 x - 1$; $\sec x\cdot\tan x\cdot dx = du$)

Try u-sub

$u = \sec x$

$du = \tan x\cdot \sec x\cdot dx$

$= \int u^8\cdot (u^2 - 1)\cdot du$

$= \int (u^{10} - u^8)\,du$

$= \cdots$

Ex $\int \cos^2(3x)\,dx$

Double-angle

$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$ — replace x with $3x$

$\cos^2(3x) = \frac{1}{2}(\cos 6x + 1)$

$= \int \frac{1}{2}(\cos 6x + 1)\,dx$

$= \frac{1}{2}\cdot\left(\frac{1}{6}\sin 6x + x\right) + C$

$= \frac{1}{12}\cdot \sin 6x + \frac{x}{2} + C$