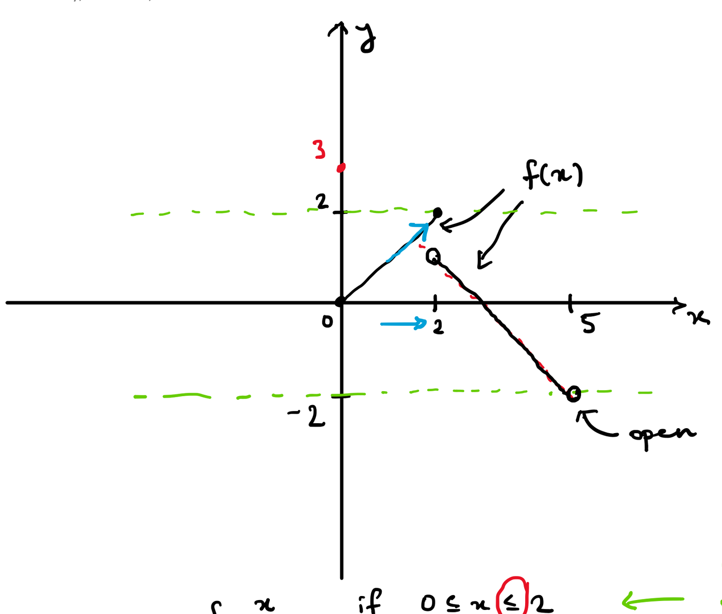
Tuesday, March 16, 2021 5:12 PM

Question 1 from practice midterm



$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 2 \\ -x+3 & \text{if } 2 < x < 5 \end{cases}$$

$y = mx + b$ (slope, y-int)
$y = x$

bracket: include 0.

Domain: all values x where $0 \le x < 5$. In interval notation: $[0, 5)$

Range: $(-2, 2]$

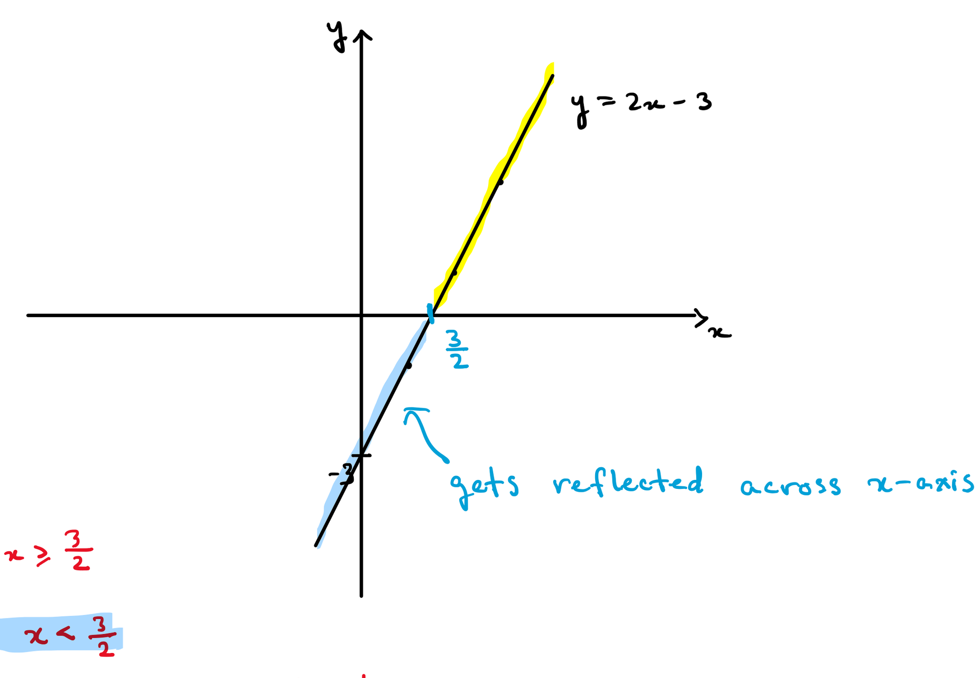
$\lim_{x \to 2^-} f(x) = 2$, $\lim_{x \to 2^+} f(x) = 1$, $\lim_{x \to 2} f(x) = \text{DNE}$

$f(2) = 2$

$y = 2x - 3$

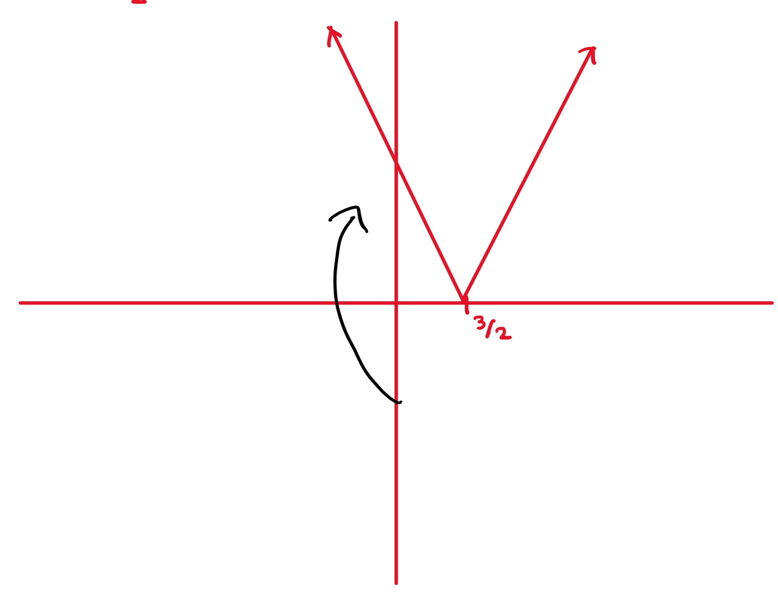
Set $0 = 2x - 3$

$x = \frac{3}{2}$

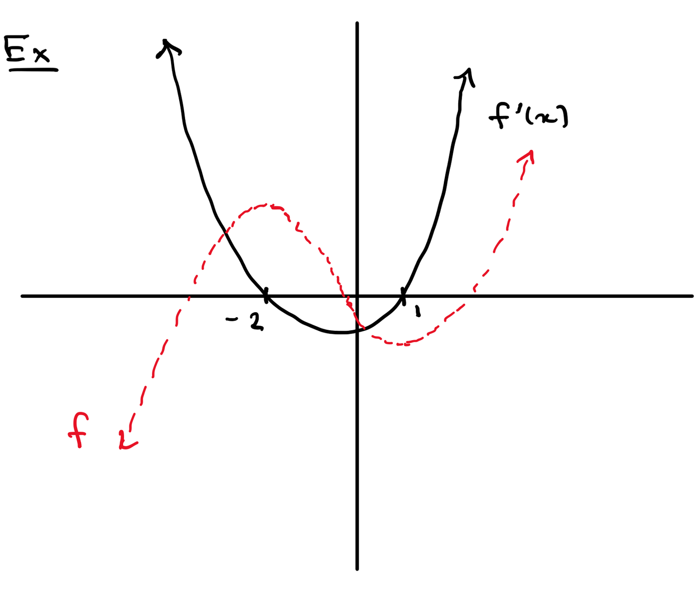


$y = |2x - 3|$

$$= \begin{cases} 2x-3 & x \ge \frac{3}{2} \\ -(2x-3) & x < \frac{3}{2} \end{cases}$$



Ex



x		-2		1	
$f'(x)$	$+$	0	$-$	0	$+$

local max

$s(t) = t^2 \cdot e^t$ ← function for position

$v(t) = s'(t) = 2t \cdot e^t + t^2 \cdot e^t$ (velocity; use product rule)

Product rule: $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$a(t) = 2 \cdot e^t + 2t \cdot e^t + 2t \cdot e^t + t^2 \cdot e^t$ (acceleration)

$= v'(t)$

9(d). $y = e^{2\cos\theta}$

Use chain rule: $y = g(h(x))$. Then $\frac{dy}{dx} = g'(h(x)) \cdot h'(x)$

$g(x) = e^x$ $\quad h(x) = 2 \cdot \cos x$

$g'(x) = e^x$ $\quad h'(x) = -2 \cdot \sin x$

$\frac{dy}{dx} = e^{2\cos x} \cdot (-2) \sin x$

$\frac{dy}{d\theta} = e^{2\cos\theta} \cdot (-2) \cdot \sin\theta$.

9(f). $y = x^2 \cdot \sin x$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ ($g(x) = x^2$, $h(x) = \sin x$)

Product rule:

$\frac{dy}{dx} = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

$= 2x \cdot \sin x + x^2 \cdot \cos x$

apply product rule to both terms

$\frac{d^2y}{dx^2} = \frac{d}{dx}(2x) \cdot \sin x + 2x \cdot \frac{d}{dx}(\sin x) + \frac{d}{dx}(x^2) \cdot \cos x + x^2 \cdot \frac{d}{dx}(\cos x)$

$= 2\sin x + 2x \cdot \cos x + 2x \cdot \cos x + x^2 \cdot (-\sin x)$

combine

$= 2\sin x + 4x \cdot \cos x - x^2 \cdot \sin x$

Ex $f(x) = 2 \cdot \cos(x^3) = g(h(x))$

Choose $h(x) = x^3$ and $g(x) = 2\cos x$

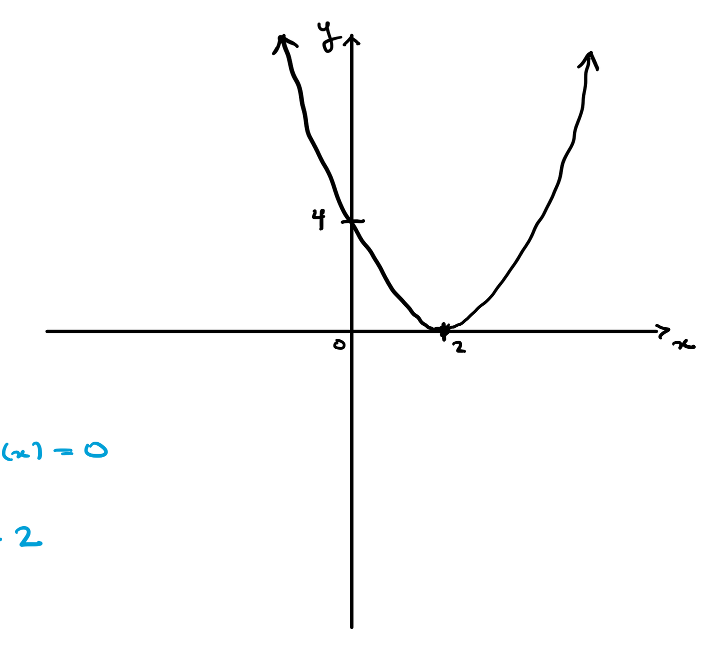
$h'(x) = 3x^2$ $\quad g'(x) = 2 \cdot (-\sin x) = -2 \cdot \sin x$

$f'(x) = -2 \cdot \sin(x^3) \cdot 3x^2$

Chain rule: $f'(x) = g'(h(x)) \cdot h'(x)$

Ex $f(x) = (x-2)^2$

$f(2) = (2-2)^2 = 0$

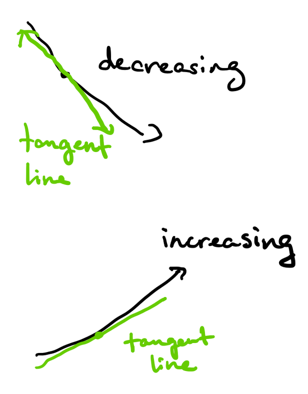


$f(0) = (-2)^2 = 4$

$f'(x) = 2 \cdot (x-2) \cdot 1$
$= 2(x-2)$

1st derivative test: Set $f'(x) = 0$

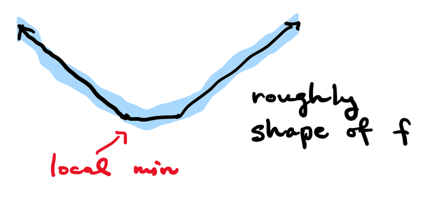
$0 = 2(x-2) \Rightarrow x = 2$



$f'(0) = 2 \cdot (0-2) = -4$

$f'(5) = 2 \cdot (5-2) = 6$

x	test 0	2	test 5
$f'(x)$	$-$	0	$+$



$f'(x) = 2 \cdot (x-2)$
$= 2x - 4$

$f''(x) = 2 > 0$ so f is concave up

concavity

Ex $\lim_{x \to 2} \frac{1}{(x-2)^2}$

Check $\lim_{x \to 2^-} \frac{1}{(x-2)^2} = \frac{1}{(\text{small} - \#)^2} = \frac{1}{(\text{small} + \#)} = \infty$

and

$\lim_{x \to 2^+} \frac{1}{(x-2)^2} = \frac{1}{(\text{small} + \#)^2} = \frac{1}{(\text{small} + \#)} = \infty$

So $\lim_{x \to 2} \frac{1}{(x-2)^2} = \infty$

What about $\lim_{x \to -\infty} \frac{1}{(x-2)^2} = \frac{1}{(\text{Large} - \#)^2} = \frac{1}{(\text{Large} + \#)} = 0$

$\lim_{x \to -\infty} \frac{4x^2}{(x-2)^2} = \lim_{x \to -\infty} \frac{4x^2}{x^2 - 4x + 4} = \lim_{x \to -\infty} \frac{4x^2}{x^2} = \lim_{x \to -\infty} 4 = 4$

Ex. $h(x) = -3x^2 + 4x$

Find eqn of tangent line at $x = 2$.

Need to find 1) a point

2) slope of tangent line.

Tangent line passes through $(2, h(2))$

$h(2) = -3 \cdot 2^2 + 4 \cdot 2 = -12 + 8 = -4$

Point: $(2, -4) = (x_1, y_1)$

Slope $= h'(2)$. $\quad h'(x) = -6x + 4$

$h'(2) = -6 \cdot 2 + 4 = -8$, $= m$

Point-slope form:

$y - y_1 = m \cdot (x - x_1) \Rightarrow y - (-4) = -8 \cdot (x - 2)$

$\Rightarrow y + 4 = -8 \cdot (x - 2)$