Q2. Given $\lim_{x\to3} f(x) = 4$, $\lim_{x\to3} g(x) = 6$, $\lim_{x\to3} h(x) = -2$

(a) $\lim_{x\to3}\left(\sqrt{f(x)} - 2\cdot h(x)\right) = \lim_{x\to3}\sqrt{f(x)} - 2\lim_{x\to3} h(x)$

$= \sqrt{\lim_{x\to3} f(x)} - 2\cdot\lim_{x\to3} h(x)$

$= \sqrt{4} - 2\cdot(-2)$

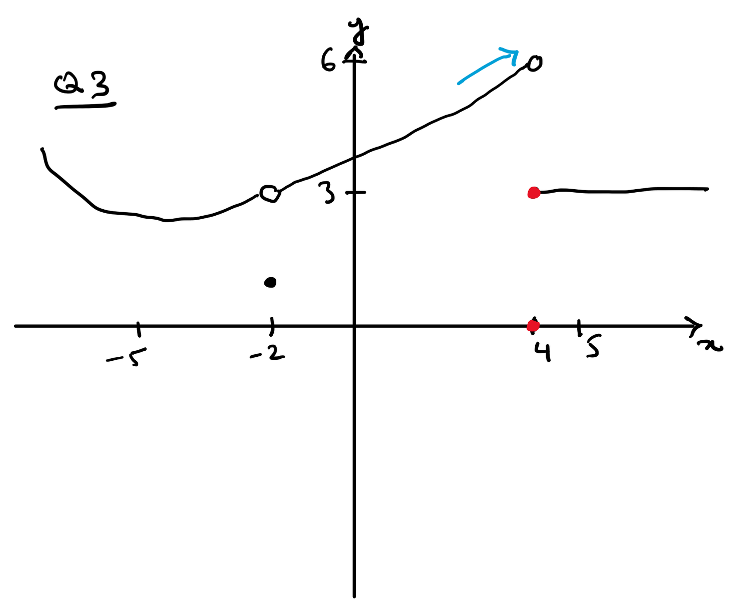
$= 2 + 4 = 6$

(b) $\lim_{x\to3}\left[g(x)\cdot(3 + h(x))\right] = \left(\lim_{x\to3} g(x)\right)\cdot\left(3 + \lim_{x\to3} h(x)\right)$

$= 6\cdot(3 + (-2))$

$= 6$

Q3



(a) $\lim_{x\to4^+} f(x) = 3$

(b) $\lim_{x\to4^-} f(x) = 6$

(c) $\lim_{x\to4} f(x) = \text{DNE}$

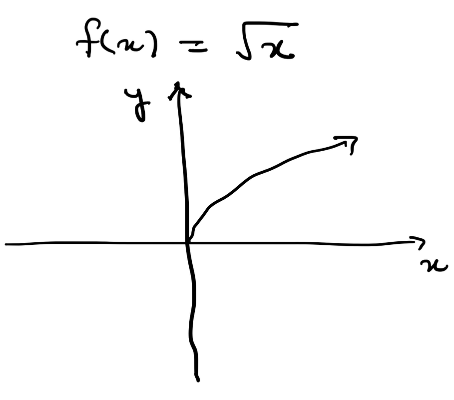
(d) $f(4) = 3$

(e) Is f continuous at $x=4$

$\lim_{x\to4^-} f(x) \neq \lim_{x\to4^+} f(x)$

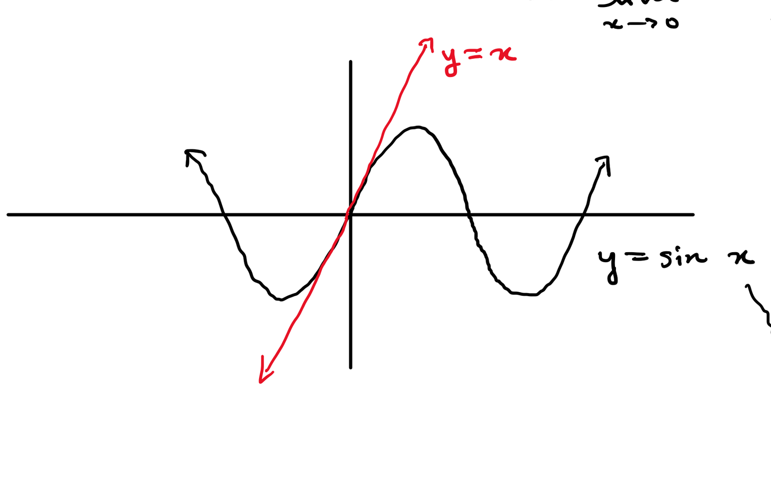
$\lim_{x\to4} f(x) \neq f(4)$

$f(x) = \sqrt{x}$



Q4 (a) $\lim_{x\to0}\left(\frac{\sin x}{x} + x\right) = \lim_{x\to0}\frac{\sin x}{x} + \lim_{x\to0} x$

$= \lim_{x\to0}\frac{\sin x}{x} = 1$



$\frac{dy}{dx} = \cos x$

$\frac{dy}{dx}\Big|_{x=0} = \cos 0 = 1$

L'Hospital's Rule

$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ ($f(x)\to 0, \pm\infty$; $g(x)\to 0, \pm\infty$)

(b) $\lim_{x\to2}\frac{x^2-2x}{x^2-4} = \lim_{x\to2}\frac{x(x-2)}{(x+2)(x-2)}$

$= \lim_{x\to2}\frac{x}{x+2} = \frac{2}{2+2} = \frac{1}{2}$

(plug in $x=2$)

(c) $\lim_{x\to\infty}\frac{15x^3 + 3x^2 + 7x + 23}{3x^3 + 5x + 11} = \lim_{x\to\infty}\frac{15x^3}{3x^3}$

$= \lim_{x\to\infty}\frac{15}{3} = 5$

(d) $\lim_{x\to4}\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to4}\frac{(x-4)\cdot(\sqrt{x}+2)}{(\sqrt{x})^2 - (2)^2}$

conjugate is $\sqrt{x}+2$

$= \lim_{x\to4}\frac{(x-4)\cdot(\sqrt{x}+2)}{x-4}$

$= \lim_{x\to4}\sqrt{x}+2$

$= \sqrt{4}+2 = 4$

(e) $\lim_{x\to2}\frac{x}{x^2-4}$ ($x\to2$, $x^2-4\to0$) $=$ DNE

$\lim_{x\to0}\frac{1}{x}$

$\lim_{x\to2^+}\frac{x}{(x+2)(x-2)} = \frac{2}{4\cdot(\text{small} + \#)} = +\infty$

$\lim_{x\to2^-}\frac{x}{(x+2)(x-2)} = \frac{2}{4\cdot(\text{small} - \#)} = -\infty$

Q5 $f(x) = \frac{1}{x} + x$

$f(x) = x^{-1} + x$

$f'(x) = -1\cdot x^{-2} + 1 = -\frac{1}{x^2} + 1$

$f'(x) = \lim_{h\to0}\frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to0}\frac{\left[\frac{1}{x+h} + (x+h)\right] - \left(\frac{1}{x} + x\right)}{h}$

$= \lim_{h\to0}\frac{\frac{1}{x+h} - \frac{1}{x} + (x+h) - x}{h}$

$= \lim_{h\to0}\frac{\frac{1\cdot x}{(x+h)\cdot x} - \frac{1\cdot(x+h)}{x\cdot(x+h)} + h}{h}$

$= \lim_{h\to0}\frac{\frac{x-(x+h)}{x\cdot(x+h)}}{h} + 1$

$= \lim_{h\to0}\frac{-h}{x(x+h)}\cdot\frac{1}{h} + 1$

$= \lim_{h\to0}\frac{-1}{x(x+h)} + 1$

$= \frac{-1}{x^2} + 1$

Q6 $f(x) = x\cdot(x^2+3)^3$ (① x, ② $(x^2+3)^3$)

$\frac{d}{dx}\left((x^2+3)^3\right) = 3\cdot(x^2+3)^2\cdot2x$

$f(x) = 1\cdot(x^2+3)^3 + x\cdot3\cdot(x^2+3)^2\cdot2x$

$= (x^2+3)^3 + 6x^2\cdot(x^2+3)^2$

$= (x^2+3)^2\left[(x^2+3) + 6x^2\right]$

Q7 $-2x+4 \le g(x) \le (x-2)^2 + 1$

Find $\lim_{x\to1} g(x)$

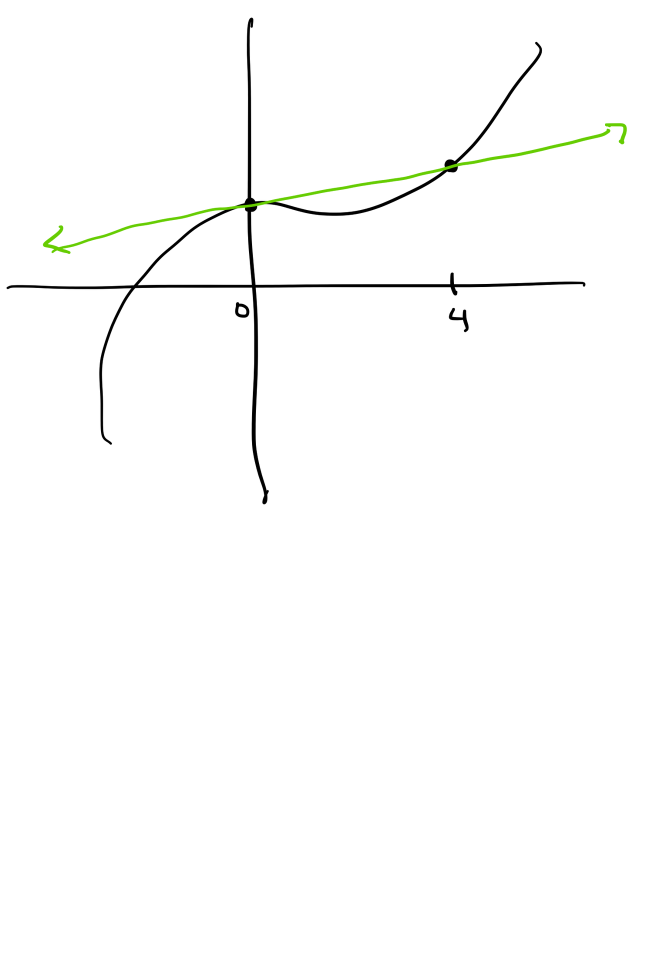
Apply squeeze thm: check

$\lim_{x\to1} -2x+4 = \lim_{x\to1}(x-2)^2+1$

$= 2$ and $= 2$ ✓

So $\lim_{x\to1} g(x) = 2$

Q8 $g(x) = \frac{x^2}{x+4} = x^2\cdot(x+4)^{-1}$



(a) Slope of secant line through pts at $x=0$, $x=4$

$m = \frac{g(4) - g(0)}{4-0}$

$= \frac{2-0}{4-0} = \frac{1}{2}$.

(b) Slope of tangent line at $x=1$

$g'(x) = \frac{(x+4)\cdot2x - x^2\cdot1}{(x+4)^2}$

$g'(1) = \frac{5\cdot2-1}{5^2} = \frac{9}{25}$

(c) Point: $(1, g(1)) = (1, \frac{1}{5}) = (x_1, y_1)$

Slope: $m = \frac{9}{25}$

$y - y_1 = m\cdot(x - x_1)$

$y - \frac{1}{5} = \frac{9}{25}\cdot(x-1)$

$y = \frac{9}{25}\cdot x - \frac{9}{25} + \frac{1}{5} = \frac{9}{25}x - \frac{4}{25}$