Nathan Chen
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MAT 125 Final Review

Here are some review problems for the final. Please send me an email if you find any errors.

1. A lighthouse is located on a small island 5 km away from the nearest point P on a straight shoreline. Suppose its light makes 8 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 2 km away from P?

2. Suppose $f(x) = \frac{e^x}{x}$.

   (a) On what interval is f increasing? On what interval is f decreasing?

   (b) Find the critical values of f and determine whether or not they are local maximum/minimum.

   (c) Find the intervals where f is concave up or down. Does f have any inflection points?

3. Consider a wall which is 50 ft long. There is a rectangular area which is enclosed by the wall and a fence (so that the fence only forms 3 sides of the rectangle and the other side is against the wall). If you have 40 ft of fencing material, what are the dimensions of the rectangle which will give you the most area?

4. Find the derivatives of:

$$y = x \ln(x) \arccos(x)$$

$$y = \frac{4}{\sqrt{3x^2 + 2}}$$

$$y = \log_{x^2}(y + 3)$$

$$2y^2 = \ln \frac{2x}{\sqrt[5]{x^2 - 5}}$$

5. At which points on the curve $y = 10 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

6. Evaluate the following limits:

$$\lim_{x \to \pi/2} \frac{\cot x}{\cos x}$$

$$\lim_{x \to \infty} \frac{2x\sqrt{x^2 + 1}}{\sqrt{x^4 - 1}}$$

$$\lim_{x \to 1} \frac{x - 1}{\sin(\pi x)}$$

7. Find the antiderivatives of

$$\frac{1}{(x+4)^3} + 2\csc x \cot x$$

$$6 + \sqrt{x^2+3}$$

Bonus: give an example of a continuous function f such that

(a) f' is always positive and f'' is always negative.

(b) f has a minimum at some point c but $f''(c)$ is not positive.

(c) f has a horizontal tangent but no local maximum or minimum.

(d) f has an inflection point but is always increasing (you can also take examples that are defined on an interval).

(e) f has both a horizontal asymptote and a vertical asymptote.