MAT 211: Linear Algebra

Practice problems

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Problem 1.
Consider a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying

$$T\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Find the standard matrix of T

Problem 2.
Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & k \end{bmatrix},$$

where k is a real parameter.

1. Find the determinant of A and say for which values of k the matrix A is invertible.
2. Find the dimensions of null(A) and col(A) as k varies.
3. For $k = 4$,
   - find the eigenvalues of A;
   - find an eigenvector corresponding to the eigenvalue $\lambda = 5$.

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Problem 3. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

1. Find the eigenvalues and corresponding eigenspaces of A. Conclude that A is diagonalizable.
2. Write down a basis $\mathcal{B} = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ consisting of eigenvectors of A. Using this, find an invertible matrix S such that $S^{-1}AS$ is a diagonal matrix.
3. Find the coordinates of $\begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix}$ with respect to the basis $\mathcal{B}$; i.e. write $\begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix}$ as a linear combination of v_1, v_2, and v_3.

4. Compute $A^{456}\begin{bmatrix}6\\-1\\-2\end{bmatrix}$.

Problem 4.

Write the matrix representing the linear transformation T on $\mathbb{R}^2$ that reflects vectors about the line $y = x$. Is it invertible? What about diagonalizability?

Problem 5.

Consider the vector subspace $W = \operatorname{span}\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right)$. Find the projection of $\begin{bmatrix}4\\2\\2\end{bmatrix}$ onto W and onto $W^{\perp}$.

Problem 6.

Find all a, b, c such that the following matrices are simultaneously non-invertible

$$\begin{bmatrix}a-4 & -2\\ b & 1\end{bmatrix}, \quad \begin{bmatrix}1 & 1\\ 4-a-c & a+b\end{bmatrix}, \quad \begin{bmatrix}1 & \frac{1}{2}-c\\ 2 & a+b\end{bmatrix}.$$

Problem 7.

Suppose v_1, v_2, v_3 are linearly independent vectors.

1. Find all scalars a and b such that

$$2av_1 - v_2 = v_1 + bv_2.$$

2. Find all scalars k such that the vectors

$$v_1 + 3v_3, \quad 2v_1 + kv_3, \quad 2v_2$$

are linearly dependent.

Problem 8. Find an orthogonal basis of $\operatorname{span}\left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\2\end{bmatrix}\right)$.