MAT 211: Linear Algebra

Practice Midterm 2

Stony Brook University Spring 2019
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Problem 1. Solve the following system of linear equations

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 10 \\ 1 & 3 & 6 & 10 & 20 \\ 1 & 4 & 10 & 20 & 35 \end{array}\right].$$

Problem 2. Give bases for $\text{row}(A), \text{col}(A), \text{null}(A)$, where

1) $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$,

2) $A = \begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}$.

Problem 3. Find all possible values of $\text{rank}(A)$ as a varies

$$A = \begin{bmatrix} 1 & 2 & a \\ -2 & 4a & 2 \\ a & -2 & 1 \end{bmatrix}.$$

Problem 4. Find all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Problem 5. Find a basis for the minimal subspace in $\mathbb{R}^4$ containing the points $(1, -1, 0, 0)$, $(0, 1, 0, -1)$, $(0, 0, -1, 1)$, $(-1, 0, 1, 0)$.

Problem 6. Find a basis for the minimal subspace in $\mathbb{R}^3$ containing the point $(0, 1, 1)$ and the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Problem 7. Let u, v be a basis for $\mathbb{R}^2$. Show that

1) $u+v, u+v$ is not a basis for $\mathbb{R}^2$;

2) $u+v, v$ is a basis for $\mathbb{R}^2$;

3) $u+v, u-v$ is a basis for $\mathbb{R}^2$.

Problem 8. Are the following transformations linear?

1) $T\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 7\begin{bmatrix} x-y \\ 3 \end{bmatrix}$,

2) $K\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ 2+y \end{bmatrix} + \begin{bmatrix} y \\ x \end{bmatrix}$,

3) $S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ |x| \end{bmatrix}$.

Problem 9. Let F be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that F reflects a vector in the x-axis. Compute the standard matrix of F.

Problem 10. Compute the determinant of

$$A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 5 & 2 & 6 \\ 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 1 \end{bmatrix}.$$

Problem 11. Is the matrix

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$$

invertible? If yes, compute the inverse of A.

Problem 12. Find all a such that the matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 1 & a & 0 \\ 0 & 1 & a \end{bmatrix}$$

is invertible.