MAT 211: Linear Algebra

Practice Midterm 1

Stony Brook University Spring 2019
Dzmitry Dudko

Problem 1. Check if $\begin{bmatrix}1\\2\\4\end{bmatrix}$ and $\begin{bmatrix}0\\6\\-3\end{bmatrix}$ are orthogonal vectors.

Problem 2. In the following problems compute $u \cdot v$.

1) $u = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $v = \begin{bmatrix}2\\-3\\1\end{bmatrix}$

2) $u = \begin{bmatrix}1\\x\\3\end{bmatrix}$, $v = \begin{bmatrix}2\\3\\y\end{bmatrix}$

3) $u = \begin{bmatrix}x\\2\\-3\end{bmatrix}$, $v = \begin{bmatrix}3\\4\\x\end{bmatrix}$

Problem 3. In the following problems find all k such that u and v are parallel vectors.

1) $u = \begin{bmatrix}k\\2\end{bmatrix}$, $v = \begin{bmatrix}2\\1\end{bmatrix}$

2) $u = \begin{bmatrix}k\\1\\k\end{bmatrix}$, $v = \begin{bmatrix}1\\2\\3\end{bmatrix}$

3) $u = \begin{bmatrix}k\\0\\-k\end{bmatrix}$, $v = \begin{bmatrix}1\\0\\-1\end{bmatrix}$.

Problem 4.

1) Find the general and parametric equations of the line passing through the points $(3, 1)$ and $(1, 0)$.

2) Are the points $(2, 1)$, $(1, 2)$, and $(4, -1)$ on the same line? If yes, find the general and parametric equations of the line passing through these points.

3) Find the parametric equation of the line passing through $(1, 1)$ and $(0, x)$.

Problem 5. Find the general and parametric equations of the plane passing through the point $(1,1,1)$ and orthogonal to the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

Problem 6. Find the general and parametric equations of the plane passing through the points $(0,1,2)$, $(1,0,1)$, and $(2,1,4)$. Does the plane also pass through the origin $(0,0,0)$?

Problem 7. Solve the following system of linear equations:

$$\begin{aligned} x + 2y - 3z &= 9, \\ 2x - y + z &= 0, \\ 4x - y + z &= 4. \end{aligned}$$

Problem 8. Solve the following system of linear equations:

$$\left[\begin{array}{cccc|c} -1 & 3 & -2 & 4 & 0 \\ 2 & -6 & 1 & -2 & -3 \\ 1 & -3 & 4 & -8 & 2 \end{array}\right]$$

Problem 9. Determine if the following vectors are linearly independent

$$\begin{bmatrix}2\\2\\1\end{bmatrix}, \qquad \begin{bmatrix}3\\1\\2\end{bmatrix}, \qquad \begin{bmatrix}1\\-5\\2\end{bmatrix}.$$

Problem 10. Find all k such that the following vectors are linearly independent

$$\begin{bmatrix}2k\\1\end{bmatrix}, \qquad \begin{bmatrix}1\\1\end{bmatrix}.$$

Problem 11. Check if the span of the following vectors is $\mathbb{R}^3$.

$$\begin{bmatrix}2\\3\\3\end{bmatrix}, \qquad \begin{bmatrix}1\\-4\\-2\end{bmatrix}, \qquad \begin{bmatrix}7\\6\\0\end{bmatrix}.$$

Problem 12. Calculate the product

$$\begin{bmatrix}2k & 1\\1 & k\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}.$$