MAT 402

Homework V

Due April 23rd. 2019. Show all your work

(1) Let f be the polynomial $f(z) = z^3 + 3z^2 + 4z + 1$. Show that f has a parabolic fixed point at -1. Find a conjugation that takes the fixed point to the origin, and hence show that f has two attracting petals.

(2) Let f be the polynomial $f(z) = -z + z^5$. Show that f has a parabolic fixed point at the origin, and that f^2 has a multiple fixed point at the origin. By calculating $f^{\circ 2}$, show that f has 4 attracting petals.

(3) Our goal in this exercise is to construct a transcendental entire function with a wandering domain. Consider first the transcendental entire function $g(z) = z - \lambda \sin(2\pi z)$, where $\lambda > 0$ is small. Show that if λ is sufficiently small, then g has attracting fixed points at the points $z_n = n$ for $n \in \mathbb{Z}$.

Now, for each $n \in \mathbb{Z}$, let U_n be the Fatou component containing z_n. (Explain why these must all be disjoint.) Let $f(z) = g(z) + 1$. Show that, for each $n \in \mathbb{Z}$, U_n is a Fatou component for f with the property that $f(U_n) = U_{n+1}$. Deduce that f has wandering Fatou components.